JOURNAL OF INFORMATION AND COMPUTATIONAL SCIENCE

JOURNAL OF INFORMATION AND COMPUTATIONAL SCIENCE

ISSN: 1548-7741

submitjoics@gmail.com

CALL FOR PAPERS GUIDELINES INDEXING

EDITORIAL BOARD HOME AIM & SCOPE

CONTACT ARCHIVES PROPOSAL

VOL-11-ISSUE-6-2021

Ashish Kumar Parashar, Assistant Professor – Department of Civil Engineering, Institute of Technology, GGV, Bilaspur, C.G., India

Sohanlal Atmapoojya – Department of Civil Engineering, K.I.T.S. Ramtek, Maharashtra, India

Page No: 122 – 141

DOI:10.12733.JICS.2021.V11I6.535569.34016

17. Real-Time Face Mask Detector

Sakshi Yelne, Rutuja Vihire, Sanket Tijare, Shweta Darne, Sumit Waware – Department of Computer Science And Engineering, Datta Meghe Institute of Engineering, Technology and Research, Wardha.

Page No: 142 – 148

DOI:10.12733.JICS.2021.V11I6.535569.34017

18. Effect of Climate Change on Status of Susceptible Hosts of Foliicolous Fungi from Katarniaghat Wildlife Sanctuary Bahraich U.P. India

Ajay Kumar, Rajiv Ranjan – Postgraduate Department of Botany M.L.K (P.G.) College, Balrampur- 271 201 (U.P.) India.

Page No: 149 – 155

DOI:10.12733.JICS.2021.V11I6.535569.34018

19. Predicting the Gross Domestic Product in India Using the Time Series ARMA Model and Policy Implications

Dr Madhuchhanda Lahiri, Assistant Professor – Department of Economics, P. N. Das College, Santinagar, Palta, West Bengal.

Page No: 156 – 169

DOI:10.12733.JICS.2021.V11I6.535569.34019

20. Mechanical modelling of Wind Turbine speed and Pitch angle for different speed profiles

C.Sucharitha, Assistant Professor – Department of Mechanical Engineering, MGIT, Hyderabad, Telangana, India

Predicting the Gross Domestic Product in India Using the Time Series ARMA Model and Policy Implications

Dr Madhuchhanda Lahiri
Assistant Professor, Department of Economics,
P. N. Das College, Santinagar,
Palta, West Bengal.
Email: macharyv20@gmail.com

Abstract

The study uses the Time Series ARIMA Model of Forecasting to predict the Gross Domestic Product at constant prices in India for two lead periods under the current pandemic situation. The order of the best ARIMA model was found to be (1,1,1). By fitting our quarterly time series data from July, 2007 (Q1 2007) to Oct, 2020 (Q2 2020), as accurately as possible, I have tried to forecast the GDP at constant prices in India for the Q3 2020 and Q4 2020. The forecast results have shown the GDP figures will fall in Q3 over the Q2 but will rise somewhat in the Q4 but will not reach the Q2 levels. So grimmer days are in the offing when we consider the macroeconomic scenario of India in the 2020-21 fiscal year. The study suggests intensive demand enhancement measures are needed to be initiated by the government to increase the consumption demand, investment demand as well as the government expenditure demand component of the aggregate demand and thereby ensure a turnaround in the GDP of India.

Keywords: *Forecasting, time series modelling, ARIMA, Gross Domestic Product, India*

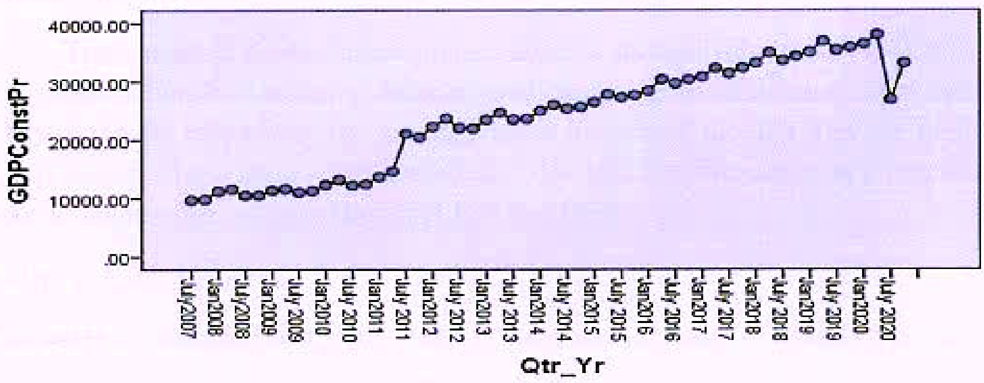
1. Introduction

The outbreak of the COVID 19 pandemic in the last months of 2019 halted the process of recovery of both the world economy as well as the Indian economy, as the corona virus is as contagious economically as it is medically. It pushed the economies the world over, in an unprecedented situation of contraction and collapse, hitherto unthought of. The sequence plot of the time- series quarterly data for the study period of 14 years (Figure 1) shows that even the world-wide financial crisis of 2007-2008 failed to produce any significant decline in the GDP figures over the period though there was a fall in the growth rate of GDP during the crisis. Overall, the GDP at constant prices in India has shown an increasing trend throughout our study period of fourteen years with the steepest rise between July 2011 and July 2012. However, Figure 1 shows a drastic and alarming fall in the GDP values between Jan 2020 to July 2020 as a result of the ensuing economic impacts of the corona virus pandemic (lockdown, travel and transport restrictions resulting in supply chain bottlenecks and demand contractions following changes in consumer motives among others). In this paper, an effort is made to forecast GDP at constant prices for the two leading quarters.

S.No.	Month, Year	GDP at Constant Prices	S.No.	Month, Year	GDP at Constant Prices
1	July 2007	9750.12	28	Apr 2014	25988.98
2	Oct 2007	9891.02	29	July 2014	25357.51
3	Jan 2008	11215.28	30	Oct 2014	25622.42
4	Apr 2008	11653.05	31	Jan 2015	26459.47
5	July 2008	10538.33	32	Apr 2015	27837.33
6	Oct 2008	10557.22	33	July 2015	27282.79
7	Jan 2009	11387.4	34	Oct 2015	27680.87
8	Apr 2009	11680.55	35	Jan 2016	28363.87
9	July 2009	11064.6	36	Apr 2016	30367.38
10	Oct 2009	11291.79	37	July 2016	29650.82
11	Jan 2010	12322.37	38	Oct 2016	30357.56
12	Apr 2010	13229.7	39	Jan 2017	30796.22
13	July 2010	12235.43	40	Apr 2017	32277.28
14	Oct 2010	12417.61	41	July 2017	31365.72
15	Jan 2011	13681.63	42	Oct 2017	32320.72
16	Apr 2011	14626.4	43	Jan 2018	33148.01
17	July 2011	21028.63	44	Apr 2018	34917.19
18	Oct 2011	20428.7	45	July 2018	33591.62
19	Jan 2012	22251.35	46	Oct 2018	34325.53
20	Apr 2012	23654..61	47	Jan 2019	35000.33
21	July 2012	22052.23	48	Apr 2019	36896.78
22	Oct 2012	21959.46	49	July 2019	35352.67
23	Jan 2013	23447.67	50	Oct 2019	35843.37
24	Apr 2013	24670.82	51	Jan 2020	36427.48
25	July 2013	23473.96	52	Apr 2020	38036.01

26	Oct 2013	23570.79	53	July 2020	26895.56
27	Jan 2014	24979.97	54	Oct 2020	33141.67

Table 1: Gross Domestic Product of India at Constant Prices (In INR Billion); NSA, 2011-12 prices



Source: Ministry of Statistics and Programme Implementation (MOSPI)

Figure 1.

Time Plot of the GDP at Constant Prices of India (INR Billion) for July 2007 – October 2020

2. Literature survey

To identify the time series, answer to the following two questions have to be found out before we proceed (Box and Jenkins, 1976): i) Whether the data are random ii) Does the series show any trend. The model developed by Box-Jenkins (1976) assumes that the underlying process is time-stationary. However, if it is not so, then Box- Jenkins recommend differencing the non-stationary time-series one or the required number of times to make it stationary. The model developed by Box-Jenkins is called the Box-Jenkins Model or Auto Regressive Integrated Moving Average Model (ARIMA Model) where the term 'I' stands for Integrated.

As'ad Mohamad studied the time-series model of peak daily electricity demand (June, 2010 – May, 2011) from New South Wales, Australia on the basis of past three, six, nine and twelve months of data. They used RMSE and MAPE to measure the forecast accurately. Alma and Ahiakpor (2011) used ARIMA model to predict inflation in Ghana with monthly inflation figures over the period 2000:6 to 2010:12. They found inflation to follow ARIMA (6,1,6) process and concluded that as inflation has a long memory and so once when the inflation is set in motion, it takes at least 12 periods (months) to bring it to a stable state. Kumar Manoj and Anand Madhu (2014) selected ARIMA (2,1,0) model for making predictions for upto 5 years for the production of sugarcane in India using a 62-years' time series data. The study also statistically tested and validated that the successive residuals (forecast errors) in the fitted ARIMA time series are not correlated and the residuals seem to be normally distributed with mean zero and constant variance. Chitra et al (2015) applied ARIMA model in supply chain

Principal P.N. Das College Palta, Bengal Enamel, N.24 Pgs management (SCM) of fresh vegetables using the SPSS software. Their analysis of monthly supply and prices of vegetables data found seasonal pattern.

3. Database and methodology

The study consists of quarterly data for the period Q3:2007 to Q2:2007 on the Gross Domestic Product in India at constant prices (in INR Billion; 2011-12 prices). The data is secondary in nature and was obtained from the Ministry of Statistics and Programme Implementation (MoSP) website. The data was analysed primarily by using the SPSS Software.

Time-series is defined as a sequence taken at successively equally spaced points in time with a natural temporal ordering. Making prediction in time-series using a univariate approach is best done by employing the autoregressive integrated moving average models ARIMA (p,d,q) model. These are a set of models that describe the process(y_t) as a function of its own lags and white-noise process (Box and Jenkins, 1974).

AR(p): $y_t = \mu + a_1 y_{t-1} + \ldots\ldots + a_p y_{t-p} + e_t$

Where μ is a constant, e_t is a white noise process and the order of the process is p.

MA(q): $y_t = a_0 + e_t + b_1 e_{t-1} + \ldots\ldots + b_q e_{t-q}$

e_t is a white noise process and the order of the process is q.

'I' stands for 'Integrated' process which shows the number of differentiations needed to make the non-stationary time-series stationary.

We follow the Box-Jenkins methodology in order to build the ARIMA model on the basis of the following steps:

i. Model Identification
ii. Parameter Estimation and Selection
iii. Diagnostic Checking or Model Validation
iv. Forecasting

The first stage of ARIMA model building is to identify whether the variable to be identified is time-stationary or not. This is done by looking at the sequence plot of the original time-series and also the differenced series. If the original series is non-stationary, then we find the order of differentiation that is needed to make the series stationary. The second stage of the identification of the model involves looking at the correlograms of the Auto Correlation function (ACF) and the Partial Auto Correlation Function (PACF) for the original time series and the differenced series. These plots are called autocorrelation functions because they show the degree of correlation with past values of the series as a function of the number of periods is the past (that is the lag) at which the correlation is computed. The ACF and PACF plot of the sample series is studied against the theoretical pattern of the corresponding correlograms as prescribed by Box-Jenkins. This involves identifying the orders determining the orders (p,q) of the AR and MA components of the model by looking at correlograms and comparing with those prescribed by Box-Jenkins. Utmost care must be taken in differencing as over-differencing will tend to increase the standard deviation rather than reduce it.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

The estimation and diagnostic checking stage involve the estimation of the parameters of the ARIMA model so identified in the last step. It also gives diagnostic statistics to judge the model adequacy, significance tests for parameter estimates help us to decide if some parameter terms are indeed necessary in the model, and Goodness-of-fit statistics help in comparing this model to other specifications of the ARIMA models identified along with.

In the forecasting stage, we forecast future values of the time series for certain specified leads and also generate the confidence intervals for the forecasted values.

4. Analysis

4.1. Identification of the parameters of the ARIMA Model

Figure 1 shows the time plot of the Gross Domestic Product in India at constant prices over our study period of Q3: 2007 to Q2: 2020. It shows clearly that the data is not time-stationary (depicting an increasing trend over time). The figure also shows that the world financial crisis of 2007-08 did not have a profound effect on the GDP of India during those years though the rate of growth fell down. But thereafter, there was a steep rise around 2011-2012 which kept on rising till the current pandemic halted the process. The GDP had a sharp decline in the Q1: 2020 and somewhat lower growth in the Q2: 2020. So now as the original series is not stationary overtime i.e., does not have a constant mean and variance, we difference the series once and check the sequence points of the new series overtime. Figure 2 shows that the first differenced time series has become stationary overtime both in its mean and variance. We need not difference it further as over differencing will result in an increase in the standard deviation.

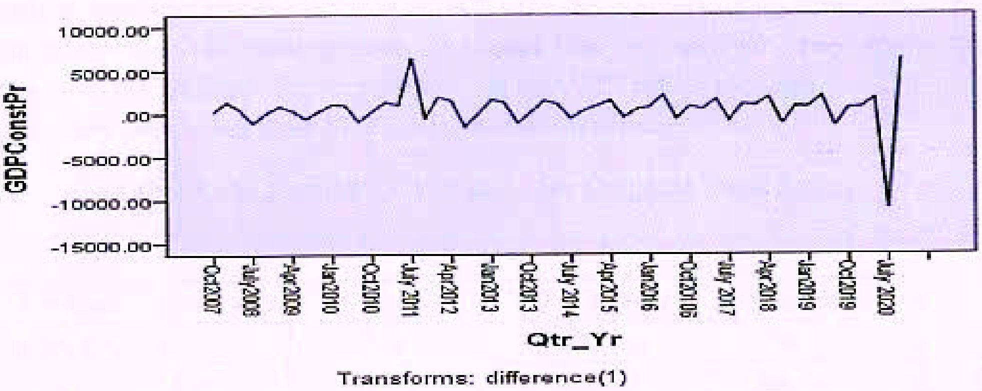


Figure 2. Sequence Plot of the First Differences of GDP at Constant Prices in India for the Period July 2007 – October 2020

Table 2. Results of Augmented Dicky Fuller Test Using the Original Time Series

Null Hypothesis: GDPCONSTPR has a unit root

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.391342	0.5795
Test critical values:	1% level	-3.562669	
	5% level	-2.918778	
	10% level	-2.597285	

We now try to find the suitable values of the (p) and (q) components of the AR and MA part of our ARIMA model (d=1 has already been identified). We can find the values of p and q by examining the correlogram (ACF) and partial correlogram of the stationary (first order differenced) time series.

Figure 3 shows the correlogram for the correlogram for the auto-correlation function (ACF) for the original series on GDP at constant prices till lag 20 and Figure 4 shows the correlogram for the partial auto-correlation function (PACF) for the corresponding time-series. A visual interpretation of the ACF plot (Figure 3) shows that the variable series under study is nonstationary as the ACF decays very slowly and the ACF values are significant till as high as 13 lags. Figure 4 shows that the PACF is also highly significant at lag 1.

Table 3. ACF and PACF Values for Original Time Series

Lag	ACF Values	PACF Values	Lag	ACF Values	PACF Values
1	0.937 (.132)	.937(.136)	11	.390(.119)	-.013(.136)
2	.908 (.131)	.246(.136)	12	.336(.118)	.020(.136)
3	.856 (.130)	-.136(.136)	13	.272(.116)	-.098(.136)
4	.807 (.129)	-.077(.136)	14	.213(.115)	-.066(.136)
5	.745(.127)	-.135(.136)	15	.154(.114)	-.027(.136)
6	.689(.126)	-.028(.136)	16	.101(.112)	.000(.136)
7	.630(.125)	-.014(.136)	17	.060(.111)	.110(.136)
8	.576(.123)	-.002(.136)	18	.023(.109)	.038(.136)
9	.509(.122)	-.118(.136)	19	-.010(.108)	-.014(.136)
10	.450(.121)	-.039(.136)	20	-.036(.106)	-.007 (.136)

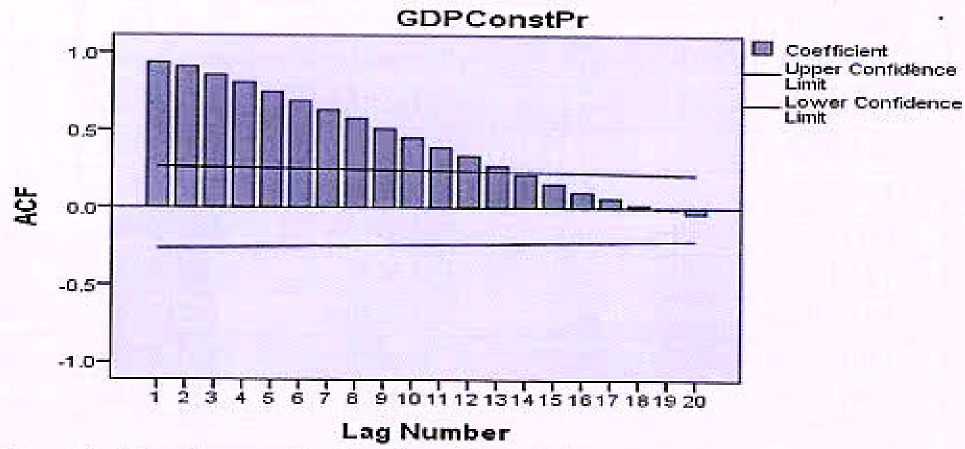


Figure 3. Correlogram for the Auto Correlation Function (ACF) for the Original Series on GDP at Constant Prices till Lag 20

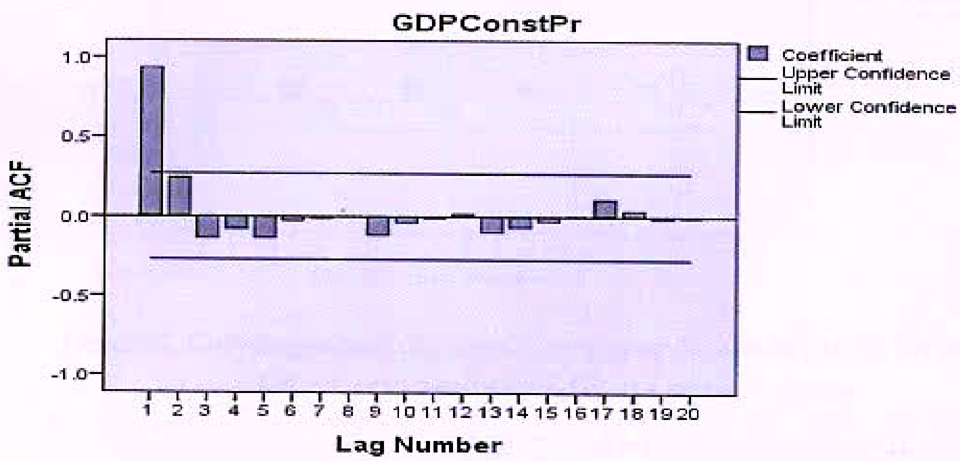


Figure 4. Correlogram of the Partial Auto Correlation Function (PACF) for the Original Series on GDP at Constant Prices

Table 4 shows the corresponding values while Figure 5 and Figure 6 show the correlograms of the ACF and PACF of the first order differenced series. On inspection of Figure 5, we see that the autocorrelation at lag1 exceed the significance limits and thereafter all the autocorrelation tails off to zero. On inspection of Figure 6, we find that the partial autocorrelation coefficient exceeds the significant limits at lag 1 and thereafter it tails off to zero.

Table 4. ACF and PACF Values for First Differenced Time Series

Lag	ACF Values	PACF Values	Lag	ACF Values	PACF Values
1	-.378 (.134)	-.378 (.137)	11	-.047(.120)	-.125(.137)
2	-.002(.132)	-.169(.137)	12	.118(.119)	.012(.137)
3	-.020(.131)	-.102(.137)	13	-.093(.117)	-.009(.137)
4	.152(.130)	.126(.137)	14	-.009(.116)	-.035(.137)
5	-.142(.128)	-.038(.137)	15	-.027(.114)	-.076(.137)
6	-.007(.127)	-.070(.137)	16	.139(.113)	.057(.137)
7	-.031(.126)	-.095(.137)	17	-.101(.111)	.009(.137)
8	.149(.124)	-.095(.137)	18	-.036(.110)	-.061(.137)
9	-.134(.123)	-.020(.137)	19	.028(.108)	-.032(.137)
10	-.018(.121)	-.071(.137)	20	.098(.106)	.042(.137)

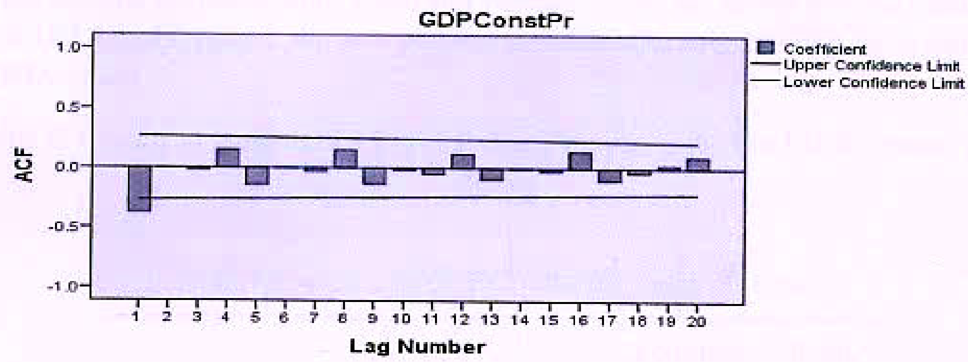


Figure 5. Correlogram of the Auto Correlation Function (ACF) for the First Differenced Series on GDP at Constant Prices

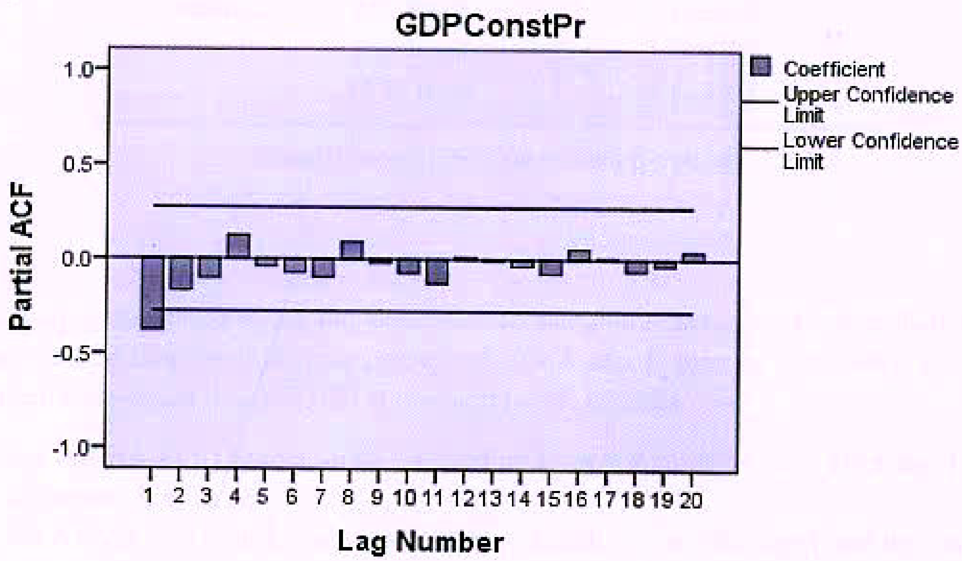


Figure 6. Correlogram of the Partial auto Correlation Function (PACF) of the First Differenced Series of the GDP at Constant Prices

4.2. Test for Stationarity: Augmented Dicky Fuller (ADF) Test

We now check our original and differenced time-series for unit root problem using the Augmented Dicky Fuller Test.

Our null hypothesis (H_0) in the test is that the time series is non-stationary and the alternative hypothesis (Ha) is that the series is stationary. On checking Table 2, we find that the p-value is greater than 0.05, so we accept the null hypothesis and conclude that the original series is non-stationary.

We now apply the ADF test on the table generated by differencing the current and previous levels of GDP (by taking the 1st differences) and obtain the results as shown in Table 5. The p-value, being less than 0.05, we reject the null hypothesis and conclude that the first differenced time-series has become stationary. Figure 2 also show that the first differenced series has become stationary in its mean and variance. Thus, we accept d=1 for identification of our ARIMA (p,d,f) model. We now proceed to the identification of our p and q components of ARIMA model.

Table 5. Results of Augmented Dicky Fuller Test using the First Differenced Time Series

Null Hypothesis: D(GDPCONSTPR) has a unit root

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-10.33588	0.0000
Test critical values:	**1% level**	-3.562669	
	5% level	-2.918778	
	10% level	-2.597285	

*MacKinnon (1996) one-sided p-values.

Comparing the patterns of the observed correlograms with those prescribed by Box-Jenkins, we find that the following possible ARIMA model may be considered for our first differenced time-series data of GDP at constant prices in India:

i. An ARMA (1,0) model since the partial autocorrelation is zero after lag 1 and the autocorrelation is zero.

ii. An ARMA (0,1) model since the autocorrelation is zero after lag 1 and the partial auto correlation.

iii. An ARMA (p,q) model where both p and q are greater than 0 since autocorrelation and partial autocorrelation both tail off to zero.

On the basis of the above considerations, we find that ARMA (1,0) and ARMA (1,1) are the best candidate models for further estimating the parameters. We observe that the lowest BIC values are for the ARIMA (1,1,0) model and hence we select this model as the best predictive model for making predictions for the future values of GDP in India.

Table 6. Coefficients with S.E., R2, RMSE, MAPE, AIC and BIC Values of the Fitted ARIMA Models

ARIMA Model	Coefficients		R2	RMSE	MAPE	BIC
	AR1	MA1				
ARIMA (1,1,0)	-0.429 (.138)	--	0.949	2018.066	5.310	15.370
ARIMA (1,1,1)	-0.130 (.139)	0.226	0.948	2073.546	5.121	15.499

Thus, we find out the parameter estimates of ARIMA (1,1,0) model and check whether the estimates are significant or not. If they are not, we omit those terms. As the ARIMA (1,1,0) model has only 1 parameter, so we obtain the following parameter estimates for our model:

$Y_t = \mu + a_1 X_{t-1} + e_t$

$\hat{a}_1 = -0.429$, $\mu = 409.839$ and

e_t is white noise with mean zero and constant variance. However, for a stationary differenced time series, the mean (μ) should be either equal to or very close to zero. Since we have $\mu = 409.839$ significant at 99% level of significance, we use the values of the mean to predict our future value.

The ARIMA (1,1,0) model is now used to forecast the future ten lead quarters of our time series data on GDP at constant prices. Table 7 shows the forecast at 95% level of significance for low and high prediction intervals.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

Table 7. 10- Quarter Forecasting for GDP at Constant Prices in India (Using ARIMA (1,1,0) Model)

Forecasted Quarter/Year	Forecasted GDP	LCL (95%)	UCL (95%)
Q3 2020	31050.29	26998.98	35101.59
Q4 2020	32532.06	27865.95	37198.17
Q1 2021	32482.51	26902.99	38062.02
Q2 2021	33089.23	26873.12	39305.34
Q3 2021	33414.69	26565.11	40264.27
Q4 2021	33860.69	26453.95	41267.44
Q1 2022	34255.03	26321.24	42188.83
Q2 2022	34671.51	26247.19	43095.84
Q3 2022	35078.51	26189.21	43967.81
Q4 2022	35489.57	26159.04	44820.09

Table 7 shows the time plot of the 10-lead quarter's forecast of the GDP at constant prices by fitting ARIMA (1,1,0) model to our time series data. Table 8 shows the corresponding ACF and PACF values for the forecast errors.

Table 8. ACF and PACF Coefficients for the Residuals/ Forecast Errors

Lags	ACF Coefficients	PACF Coefficients	Lags	ACF Coefficients	PACF Coefficients
1	-.034(0.137)	-.034(.137)	11	-.030(0.148	-.075(.137)
2	-.073(0.138)	-.074(.137)	12	.090(0.149)	.057(.137)
3	.035(0.138)	.030(.137)	13	-.074(0.150)	-.020(.137)
4	.132(0.138)	.130(.137)	14	-.062(0.150)	-.044(.137)
5	-.116(0.141)	-.104(.137)	15	.017(0.151)	-.024(.137)
6	-.081(0.143)	-.074(.137)	16	.130(0.151)	-.085(.137)
7	.012(0.143)	-.016(.137)	17	-.088(0.153)	-.038(.137)
8	.124(0.143)	.111(.137)	18	-.083(0.154)	-.065(.137)
9	-.115(0.145)	-.078(.137)	19	.051(0.155)	.007(.137)
10	-.101(0.147)	-.093(.137)	20	.121(0.155)	.072(.137)

Figure 7 gives a plot of the fitted and the forecasted values using the estimated ARIMA model. We finally study the distribution of the forecast errors (residuals) obtained on fitting our ARIMA (1,1,0) model by plotting them. Figure 8 shows the plot of the forecast errors (residuals) of the fitted ARIMA (1,1,0) model which shows that they are roughly constant in their means and variances overtime (although there exists a substantially larger variance at the end of our estimation period). Figure 9 shows the histogram of the residuals and the curve fitted to it seems to follow the normal distribution to a great extent. The normal Q-Q plot of the residuals in the fitted ARIMA model is shown in the Figure 10 which also seems to confirm that the error terms are normally distributed.

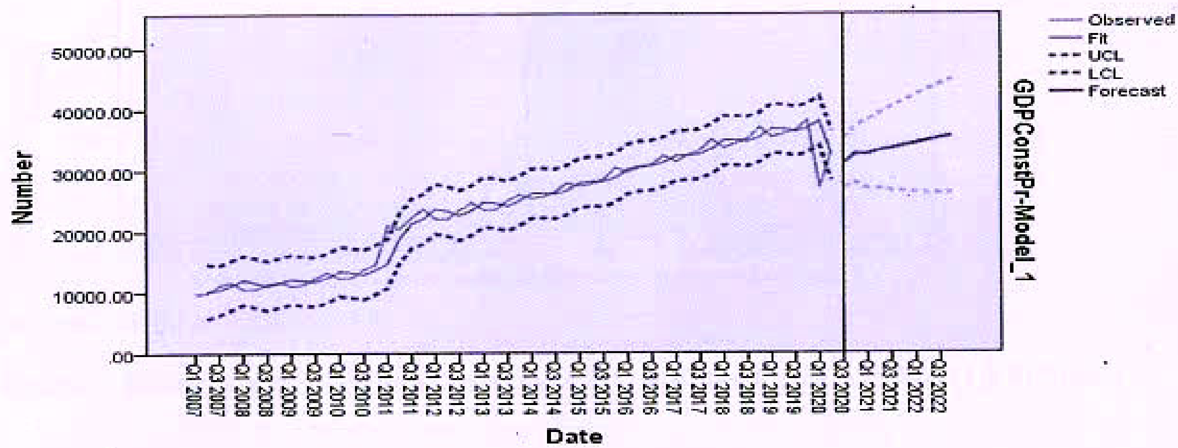


Figure 7. Fitted and Forecasted Values of GDP at Constant Prices

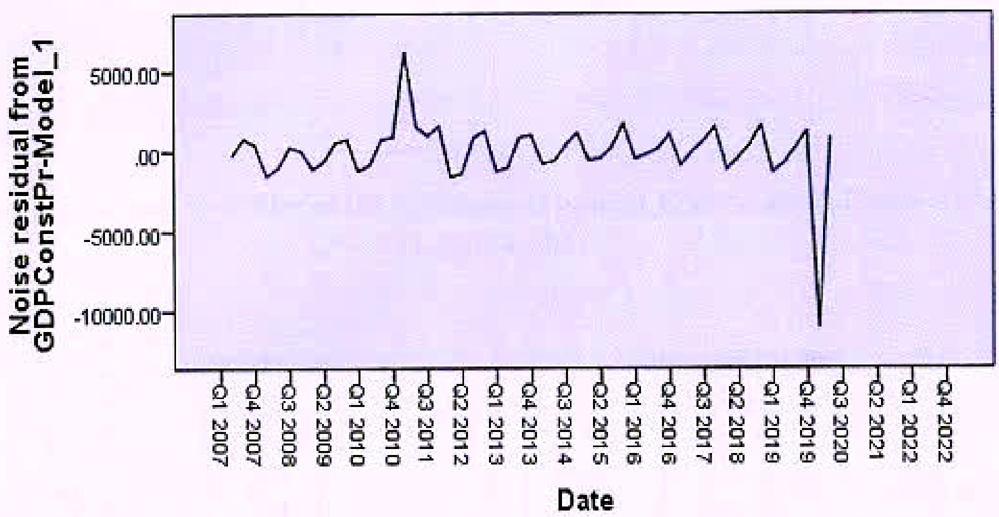
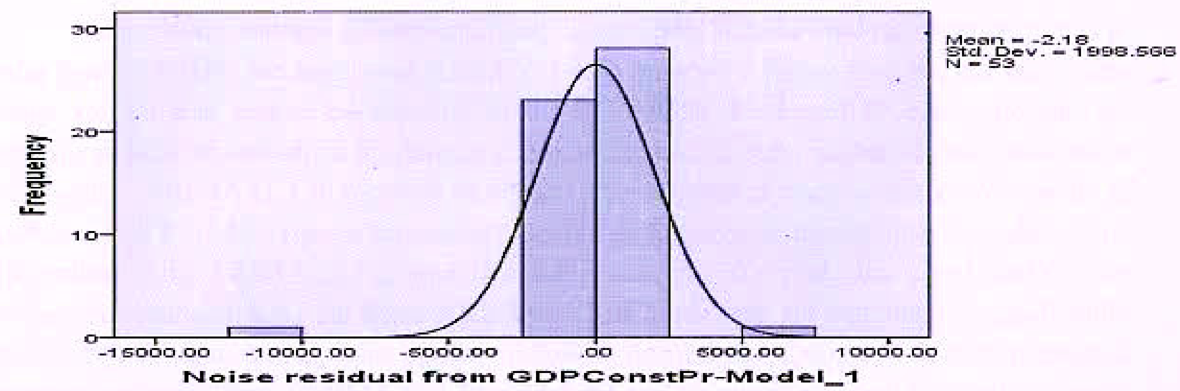


Figure 8. Plot of Residuals (Forecast Errors) of the Fitted ARIMA (1,1,0) Model

Figure 11 shows the correlogram of ACF and PACF of the forecast errors and the corresponding values with their S.E. are given in Table 8. On an examination of the Figure 11, we find that all ACF and PACF coefficients are statistically insignificant and we can conclude that there is no evidence for non-zero autocorrelationsin the forecast errors at lags 1 to 24 in



our fitted ARIMA (1,1,0) model.

Figure 9. Histogram of Residuals (Forecast Errors) of the Fitted ARIMA (1,1,0) Model

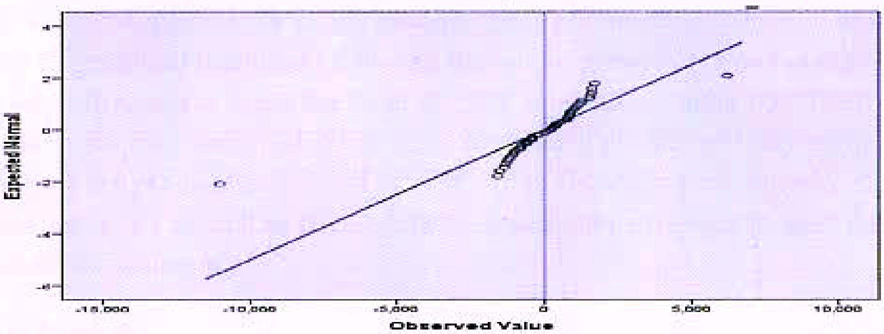


Figure 10: Normal Q-Q Plot of the Residuals (Forecast Errors) of the Fitted ARIMA (1,1,0) Model

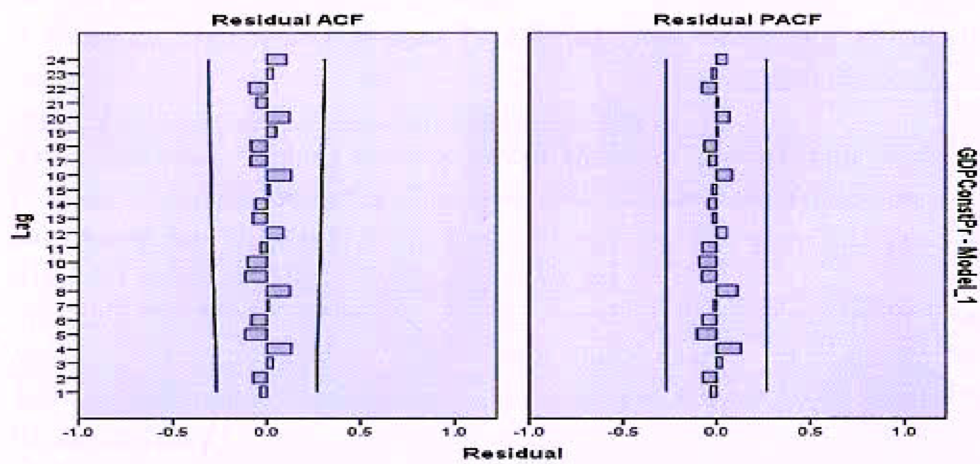


Figure 11. ACF and PACF of Residuals (Forecast Errors) obtained on fitting ARIMA (1,1,0) Model

5. Conclusions

In our study, using a 13-year quarterly time- series data of GDP at constant prices pf India from 2007-20, we have used ARIMA (1,1,0) model for forecasting the corresponding values for 10 lead quarter-periods till 2020: Q4., amidst the Covid 19 pandemic and the subsequent lock downs which produced a profound impact on the macroeconomic variables of the country. ARIMA (1,1,0) model was chosen after a detailed study of the correlograms of ACF and PACF of the original time-series and the differenced series and also considering the BIC values of the ARIMA (p,d,q) models initially selected. The study also concluded that the successive residuals from the fitted ARIMA (1,1,0) model are not statistically significantly auto-correlated and the residuals appear to follow normal distribution with zero mean and constant variance. Thus, ARIMA (1,1,0) model appears to provide a good forecasting model for the GDP in India.

The ARIMA (1,1,0) model predicted a gradual increase in the GDP at constant prices for all the 10 quarters for which forecasts were obtained but there will be noteworthy fall in the rate of growth of predicted GDP over the rate of growth obtained in the Q2 of 2020. The GDP level will nowhere reach the level of GDP attained by India (Q420219) when the Covid 19 pandemic struck India (Q1 2020). The slow rate of growth will take many years for the country to reach the pre-pandemic level of GDP. Thus, drastic macroeconomic stimulus in the form of both monetary as well as fiscal stimulus needs to be provided by the GoI to pull the economy out of this situation.

References

1. *G.E.P. Box and G.M. Jenkin, "Time Series of Analysis, Forecasting and Control", San Franscico, Holden Day, California, USA.* ***(1976).***
2. *G.M. Ljung and G.E.P. Box "On a measure of lack of fit in time series models", Biometrika, Vol. 67,* ***(1978)****, pp. 297-303.*
3. *A. Coghlan, "A Little Book of R for Time Series", Readthedocs.org, (2010), Available online at: http://a-little-book-of-r-for-time-series.readthedocs.org/en/latest/src/timeseries.html*
4. *As'ad, Mohamad "Finding the Best ARIMA Model to Forecast Daily Peak Electricity Demand", Applied Statistics Education and Research Collaboration (ASEARC) Conference Papers, 12* ***(2012)***
   Available online at: https://ro.uow.edu.au/asearc
5. *N Chitra, Ra Shanmathi and R. Rajesh, "Application of Arima Model using SPSS Software - A Case Study in Supply Chain Management"' International Journal of Science, Technology & Management, www.ijstm.com, Volume No 04, Special Issue No. 01,* ***(April 2015)***
6. *M. Kumar and M. Anand, "An Application of Time Series ARIMA Forecasting Model for Predicting Sugarcane Production in India", Studies in Business and Economics 9(1)* ***(April 2014)****, pp.81-94. Available online at: https://www.researchgate.net/publication/263505554_An_Application_Of_Time_Series_Arima_Forecasting_Model_For_Predicting_Sugarcane_Production_In_India*

Principal
P.N. Das College

JOURNAL OF INFORMATION AND COMPUTATIONAL SCIENCE

JOURNAL OF INFORMATION AND COMPUTATIONAL SCIENCE

ISSN: 1548-7741

submitjoics@gmail.com

VOL-11-ISSUE-5-2021

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs

Challenges Faced By Women In Higher Education In India

Bonny Samanta*

Assistant Professor, P.N.Das College, North 24 Parganas, West Bengal, India
E-mail: bonnysamanta@pndascollege.in

&

Rimpa Saha

Assistant Professor, Dewan Abdul Gani College, Dakshin Dinajpur, West Bengal, India
E-mail: rimpasaha456@gmail.com

Abstract

Education is one of the significant instruments for overall development of any society. India has the third largest higher education system in the world, next to United States and China. The main governing body at the tertiary level is the University Grants Commission. There are approximately 864 universities (includes State, Deemed to be University, Central Private University) approximately 39000 colleges spread across the country. India is a developing country and continuing to develop in the field of education. There are many challenges or problems in Indian higher education system, and also have lot of opportunities to deal with this problems and make education much better. An important task of higher education is to develop human resource. Education plays an important role in empowering women. Higher education encourages and empowers women to play vital role in social development as well as country. In case of higher education, women are facing personal and social problems which inhibit the progress of women in higher education. Great numbers of girls especially in rural areas drop-out before they reach higher education. Before understanding their potentials some burdens are imposed on them and as a result, drop-out rate increases. Women's empowerment can be strengthened through higher education. Higher education enables qualified women to become an ideal in society and motivates them to become a role model for other women. This paper highlights the problems in higher education faced by women in India and some suggestions to overcome these problems.

Key words: *Higher Education, development, empowerment, problems*

Introduction

"There is no chance of the welfare of the world unless the condition of women is improved. It is not possible for, a bird to fly on one wing"... Swami Vivekananda.

Education is required in every aspect of human life. Over time, the speed of communication has increased, and with it technological developments and innovations, people and society have been trying to cope with this unprecedented development. Nowadays education has outlived passed over its traditional methods and is moving towards the virtual and e-learning. These techniques in encourage students to adopt for self-learning. Higher education in contemporary times foster and encourage inspired the students in self-learning among student. The changing world is a universal topic of interest, especially its resonance to higher education. The role of higher education is immensely complex and essential in today's world. College and universities research is being influenced by the current situation. Education is the one and most powerful instrument of social change. Each and every sector of social development depends on human resources whose efficacy depends on the overall robustness of the education system.

The Indian higher education system is the third largest in the world, next only to that of the United States and China. According to the report of All India Survey Higher Education 2016-17 there were 864 universities(Central Universities, Deemed, State Universities, Private Universities), 40026 colleges and 11669 standalone institutions(listed for AISHE 2016-17) and according to All India Survey of Higher Education 2017-18 there are 903 universities (State Universities, Deemed to be Universities, Central Universities, Private Universities), 39050 colleges and 10011 standalone institutions (listed for AISHE 2017-18) in India. The main governing body at tertiary level is University Grant Commission (India), which sets the standard for higher education, advises the governments, and helps coordinate educational service and reform between the Centre and the State.

As per **AISHE 2016-17**, most of the colleges run only undergraduate level programme, only 2.6% of colleges run Ph.D. level programme and 36.7% of colleges run post-graduate level programme, 59.34% colleges are located in rural area and only 9.3% colleges are exclusively for girls and according to **AISHE 2017-18**, 36.7% of colleges run post-graduate programmes and 3.6% of colleges run Ph.D. level programme, 60.48% colleges are located in rural area and 11.04% colleges are exclusively for girls.

India is the second most populous country in the world, it has substantial human resources. In terms of social, economic prosperity India promising human resources act as a harbinger of progress. According World Bank, India has the potential to become the human resource capital of the world. According to the gender statistics revealed by Census 2011, 52 percent of the population is male and nearly 48% female. The advancement of society is not possible by men alone, therefore the opportunities for women to come forward need to be enriched, only then will the society develop and prosper. Women not only constitute valuable human capital of the country, but their development in education and socio-economic field also sets the pace of strong economical growth of the country. The development of every aspect of the society will remain a dream forever if it neglect women's education .Women have played a major role in developing ideas and working towards a better future for the world . They have a achieved special recognition in all areas imaginable and the field of education is no exception .The problem is that women's population is still considered to be the second largest citizen of the country.

Women empowerment can be strengthening through higher education. Higher education empowers qualified women to become an ideal of society and motivates them to become a role model for other young girls. It is a matter of great challenge to provide higher education to a girl in our country. In the past, girls were not allowed to study. Many obstacles in society stood in the way of progress. Over the years, the progress of women's education has substantially improved, but the barriers have not been completely eradicated. Although girls are now being enrolled in higher education, but challenges hindering their progress still remain.

- **Objectives**

The objectives of the paper are as follows:

- ❖ To discuss the present condition of literacy in higher education of women.
- ❖ To identify the problems and challenges facing by women in higher education.
- ❖ Finding solutions to overcome of these challenges

- **Methodology**

This study is qualitative in nature; where data have been collected from secondary sources like various published records, reports, books and websites etc. The students' enrollment rates have been taken from All India Survey on Higher Education (AISHE), 2016-17 and 2017-18. The population data have been taken from Census 2011. And other data are taken from A statistical analysis of child marriage in India (based on Census, 2011), Report of Radhakrishnan Commission, UGC (Consolidated list of College and University).

- **Finding**

According to the objective of the study findings are discuss bellow:

❖ **To discuss the present condition of literacy in higher education of women**

Table 1. Literacy Rate

Total Literacy Rate(in terms of total population in India)	Female(in terms of total population of women)
74.04%	65.46%

Source: Census 2011

Table 2. Enrollment In Higher Education (Academic year 2016-17 and 2017-18)

Undergraduate level				
Programmes	**Total Number of Enrollment(2016-17)**	**Female Enrollment(2016-17)**	**Total Number of Enrollment(2017-18)**	**Female Enrollment(2017-18)**
Arts(B.A)	97.3 lakh	53%	95.06 lakh	52.8%
Science (B.Sc.)	46.8 lakh	48%	48.19 lakh	49.3%
B.Tech.	21.7 lakh	27.2%	21.19 lakh	27.62%
B.E (Bachelor in Engineering)	19.1 lakh	28.9%	18.2 lakh	28.75%
Commerce(B.Com)	39.9 lakh	47.5 %	40.13 lakh	47.54%
Medical	9.8 lakh	6 lakhs	10.99 lakh	6.7 lakh
Management	5.6 lakh	2.2 lakh	5.97 lakh	1.2 lakh
Law	3.5 Lakhs	1.1Lakhs	3.7 lakh	1.2 lakh

Source: AISHE (All India Survey on Higher Education) 2016-17, 2017-18

Post-Graduate Level				
Streams	**Total Number of Enrollment(2016-17)**	**Female Enrollment (2016-17)**	**Total Number of Enrollment (2017-18)**	**Female Enrollment (2017-18)**
Agriculture and allied course	20,732	33.5%	26,614	35.2%
Commerce	4.47 lakh	60.1%	4.59 lakh	58.9%
IT and Computer Science	2.20 lakh	48.3%	2.1 lakh	48.8%
Foreign language	2,06,320(in English)	66.5%	2,03,5879(English)	62.85%

Indian language	3.34 lakh	1.24 lakh	3.28 lakh	1.25 lakh
Medical Science	1,33,329	45%	1,44,694	57.1%
Home Science	11,484	10,345	Unknown	Unknown

Source: AISHE (All India Survey on Higher Education) 2016-17, 2017-18

Ph.D.				
Stream	Total Number of Enrollment (2016-17)	Female Enrollment (2016-170	Total Number of Enrollment(2017 -18)	Female Enrollment (2017-18)
Agriculture and allied course	5,379	39.4%	5,612	41.1%
Commerce	4,146	47.1%	4,493	53.35%
Science	6,786(Chemistry)	2,583	7562(Chemistry)	3,083
	5,241	1,930	5,932(Physics)	2,127
	3,335	1,427	3,894 (Mathematics)	1,751
Social Science	2,414 (Economics)	1,106	2,990 (Economics)	1415
	1,448(Political Science)	562	1,920(Political Science)	734
	1,433	763	1,879(Sociology)	1,010
IT and Computer	2,458	49.3%	2,512	52.2%
Foreign Language	2,609(in English)	57.7%	3,110 (in English)	60.7%

	32(in French)	59.4%	67(in French Language)	49.25%
Indian Language	1,888(in Hindi)	8,955	2,270(in Hindi language)	1,096
	939(in Sanskrit)	413	971(in Sanskrit)	442
Engineering and Technology:	4,553(Mechanical)	8.7%	5,349 (Mechanical)	8.9%
	4,183(Computer Engineering)	39.3%	5,235(Computer Engineering)	40.16%

Source: AISHE (All India Survey on Higher Education) 2016-17, 2017-18

❖ **To identify the problem and challenges facing by women in higher education**

- **Challenges:**

Women are facing many challenges in higher education, such as gender disparity, conservative mindset, sexual harassment, lack of parental awareness, poor economic conditions, early marriage, burden of responsibility of households, lack of transportation, lack of women colleges and universities in rural and other backward areas etc.

- **Gender disparity:**

Gender inequality in higher education is a persistent problem in the Indian society. The reality of gender inequality in India exits in every field like: education, employment, opportunities, health, cultural issues, social issues, economical issue etc. Inequality in access to higher education result in socio-economic inequalities, in the society which, in turn, accentuate to access to higher education result in inequality in access to labor market, resulting in inequalities in earnings contributing in turn to socio-economic and political inequalities (Karak and Sen, 2017). The female access to higher education is lower in India. The enrolment of men is larger than that of women. Over the past decades, gender disparity has declined in various levels of education in India, but it needs to be further. [17] A progressive change in social attitude can bring about positive changes in higher education.

- **Conservative mindset towards the higher education of women:**

The present modern society has observed many changes and numerous developments in different fields like: economic, social, cultural, science, information, communication technology etc., but unfortunately very little changes are seen in the conservative mindset of people. Although India is a developing society, women are still facing criticism of dress code, appearance life style etc. People with conservative mindset people cannot easily accept women's liberation in education. In the name of tradition impose the burden of prejudice on women, and it affects them a girl mentally as well as physically. Subjected to this stigma and deeply affected by it many girls are often forced to abandon their dreams of freedom and higher education. [26]

- **Early marriage**

Early marriage often means the end of education for girls. It is not always true that girls drop-out of schools or higher educational institutes because such educational institute are inaccessible or expensive, far from their home or lack of parental awareness. Some parents are thinks that marriage is the best option for their daughter. Before completion of the stage of specific education, some girls are forced to marry either by their parents or by the pressure and influence of communities they belong to. Unfortunately there are quite a few practical obstacles on married girls' way back to educational institute, as a result, as far as higher education is concerned their path to progress become increasing difficult and challenging .

- **Lack of transport facilities:**

Women in rural, tribal, hilly and remote areas women in India face many problems in accessing educational institutes. Due to a very limited access to transport services, it is difficult for women to go far away to study. For the inconvenience of transport facility, they are often forced to discontinue their study. In urban area it has become an obstacle to the education of many girls' due public transport inadequacy, overcrowded public transport, intense traffic, growing number of accidents and fatalities etc. If educational institutions are not available nearby, travelling for a girls may become major obstacle.

- **Sexual harassment in institute offering higher education**

It was informed by the Ministry of Human Resource Development to the LokSabha in March, 2018, that cases of sexual harassment cases in various higher educational institutes have increased 50% in the 2016-17 academic session from the 2015-16 academic session.[14] Prevention of sexual harassment in colleges and other institutes of higher education is an alarming pressing issue that needs to be addressed urgently. The University Grants Commission notified India's first gender neutral regulations (Prevention, prohibition and redressal of sexual harassment of women employees and students in higher educational institutions) Regulation, 2015. This regulation describes the responsibilities of higher educational institutions. This act is 'decisively against all gender based violence perpetrated against employees and students of all sexes, recognizing that primarily women employees and students, some male students and students of the third gender are vulnerable to many forms of sexual harassment and humiliation and exploitation.'[24]

Girls who are sexually harassed in higher education institute, they do not want to attend the educational institute due to fear of harassment, bullying, and sometimes they leave their studies for this reason. Such kinds of behavior towards a girl disrupt her physical and mental state and shows frustration and depression, which creates obstacle to her progress in education.

- **Parental attitude towards girl's higher education:**

Thanks to our parents who enables us to obtain higher education. Those parents who have attained better education are more empathic. They understand the value of education especially the necessity of higher education for females in a society that has for centuries harbored patriarchal values and tradition. Such parents are characterized by a more positive attitude towards education, on the other hand in rural and other remote areas people who are less educated, and have negative attitude, prevents women from adopting higher education. Educated parents do not discriminate between sons and daughters. They do not subscribe blindly to patriarchal values and social norms that considered women to be inferior to men. On the contrary they believe in gender equality which goes a long way in securing the future of the country.

- **Burden of responsibility of households**

Traditionally it is expected that women will bear the most of the responsibility in the family. The burden of predetermined responsibility is imposed on them. Especially in orthodox families, pursuing higher education become an obstacle for girls – even if it is possible for women to pursue higher education, they have very little time for themselves after meeting the needs of the family.

- **Lack of girl's College especially in rural and tribal areas**

The number of women's higher educational institution exclusively for women is much less compared to those institutions which are engaged to promote and facilitate education among male students. Therefore in many cases, particularly in remote areas often obligate to travel long distance to reach the educational institutions that are either gender neutral or are meant exclusively for them. Many people are still not comfortable with the idea of co-education, condemning and censuring the concept as socially detrimental and, even worse ,morally dangerous . They think that corruption is bound to breed when both girls and boys are study together in the same institution, so they do not agree to send their daughter to co-education institutions.[16]

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

- **Economical problem**:

Another major issue in higher education is economic problem. Most of the people in India are either poor or belong to the middle class. They work hard to meet the basic needs of their families. Spending money to provide higher education to their children becomes a problem for them. Despite having a positive attitude toward higher education of their girl child, they are often helpless and unable to provide higher education to their daughters owing to financial constraints.

- **Forced labour:**

Force labour can be understood as work that is performed in voluntarily and under the menace any penalty. It refers to a situation where a person are coerced to work through the use of violence or intimation, or by more subtle means such as accumulated debt, retention of identity papers or threats of documentation to immigration authorities.[23]

According to the ILO(International Labour Organization) Force Labour Convention, 1930(No.29), force or compulsory labour is: "all work or service which is exacted from any person under the threat of a penalty and for which the person has not offered himself or herself voluntarily" [28]

Forced labour may be imposed on adults, children and women by state authorities, by private enterprises or by individual players. It is seen in all types of economic activity, such as housekeeping, construction, agriculture, manufacturing, sexual exploitation, forced begging etc. and in every country. [28]

Some girls, from poor families are often forced to participate in compulsory labour. Sometimes they are unable to complete or continue their education and fail to adjust their work with education.

❖ **Find out the solution to overcome the problems**

In order to increase the number of enrollment of girl students, amenities and decencies have to be increased in higher education institutions.

- Increase number of female teachers in co-education institutions (ShanjhnduNath, 2014).
- Increase number of colleges and universities, so that women can enroll in higher education and help them to complete their higher studies.
- Parents should be more aware of the participation in higher education of their girl child.
- Provide counseling for parents, other family members and people from the secondary stage of education (ShanjhnduNath, 2014).
- The transport facilities need to improve in rural and tribal areas (ShanjhnduNath, 2014).
- Protective measure need to be institutes and implemented for women to feel safe inside and outside of their institution.
- Introduce special scholarship to encourage poor and meritorious female students to pursue higher education.

- Setting up a special policy for equal right in higher education and establish commission for it (ShanjhnduNath, 2014).
- Establish a greater number of institutes providing in rural, tribal and underdeveloped areas.
- Increase and improve halls of residence and hostel facilities.
- Introduce fellowships, stipends and grants to encourage women's in research (ShanjhnduNath, 2014).
- Government should formulate and implement various policies to reduce drop-out rate of female students (ShanjhnduNath, 2014).
- Increase vocational training institutions for promoting the entry of women in higher education (ShanjhnduNath, 2014).
- Early marriage is one of the major obstacles that prevent women from accessing higher education, it must be stopped and if required counseling for the parents of female students should be arranged (ShanjhnduNath, 2014).
- Conservative attitude toward women education the education of women must be removed from the society.
- Increase women's confidence.
- Establish a greater number of distance and open learning centers for enrollment in higher education of rural women students.
- Provide bank loan on friendly terms and conditions.
- Introduce attractive projects and grants to encourage women in research.
- In order to provide higher education, gender biasness must be removed from the families.
- Introduce skill based higher education (ShanjhnduNath, 2014).
- Provide seamless internet facility and connectivity in rural, tribal and other underdeveloped areas.
- Institutes of higher education need to take firmer and more decisive steps to stop sexual harassment of female students and employees.
- Empowered women in India through quality education and skill-based education.
- Increase more job opportunities.
- Create institutional campus free from discrimination, harassment, exploitation, retaliation at all level.[24]
- Colleges and other institutes providing higher education must take necessary action for any misconduct in their respective institutions.

- **Conclusion:** The onus of change is not just on the government but on every individual. The question of female literacy is a highly relevant one in contemporary India and calls for a reformed mindset with regard to the role of women society .It must be understood that ignoring the relevance and need of female literacy can never launch a society or country onto the path of progress .In fact downplaying the importance of female literacy can seriously hamper the prospect of a country .On the contrary , we should devise strategies to catapult our country into the path of development and progress .In this context ,it may be said that "each one teach one" is useful approach to adopt .With a view to change the present situation of women who, owing to a host of reason ranging from socio-culture to economic ,are still deprive of the benefits of education ,we must be already to bring about the necessary changes

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

in the society and most importantly ,in the mind set of the people .We should pledge to promote the general welfare of women and facilities education among the less-privileged female population of the country for as swami Vivekananda said the world cannot prosper unless the situation of women is improved.

References

[1] Barman, S (2015).Policies and recommendation of women education in the context of modern Indian History. International Journal of Applied Research, 1(9)786-788. Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=http://www.allresearchjournal.com/archives/2015/vol1issue9/PartL/1-9-66.pdf&ved=2ahUKEwjLlZSHzcjhAhXiKqYKHc3kC50QFjABegQICBAB&usg=AOvVaw3in-MIdy96YbQfIbjExJ9T&cshid=1555005290287_on April 11,2019.

[2]Chakraborty, S (2013).Higher Education for Women in India: Some Achievement and Challenges. International Indexed & Refereed Research Journal, 3(40) Retrieved from https://www.academia.edu/6225196/HIGHER_EDUCATION_FOR_WOMEN_IN_INDIA_SOME_ACHIEVEMENTS_AND_CHALLENGES_on April 22, 2019.

[3]Ghara, T.K (2016).Status of Indian Women in Higher Education. Journal of Education and Practice, 7(34):5864.Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://files.eric.ed.gov/fulltext/EJ1126680.pdf&ved=2ahUKEwiH1rClg_HhAhUBro8KHRN1BekQFjAAegQIAxAB&usg=AOvVaw0Tqk7D1NlosLhq_TKSmCs2on April 16, 2019.

[4] Jahan, K. &Selvarani, D.C. (2015). Higher Education in India: Issue and Challenges. International Conference on Humanities, Literature and Management, Jan 9-10: 81-86. Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://icehm.org/upload/2797ED0115098.pdf&ved=2ahUKEwilyrWOydLhAhXMto8KHbU9DLAQFjACegQIBRAB&usg=AOvVaw12wB6nPAmWGlDmHX_THUY2 on April 15, 2019.

[5] Karan,S.(2017). Women Education in India & its Importance: An Analytical Study. International Journal of Humanities & Social Science, 4(1): 278-287. Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://www.ijhsss.com/files/29_7o32q5zh.-Suparna-Karan.pdf&ved=2ahUKEwj5keCNhMvhAhWHuo8KHR0BDsQQFjAAegQIARAB&usg=AOvVaw00KnUOOwmT9lwUFeqGFxIP on April 13, 2019

[6] Karak, S& Sen, K. (2017). A Study on Gender Inequality in Higher Education in Indian Context.PANCHAKOTesSAYS, 8(2):111-115. Retrieved from https://www.academia.edu/37245430/A_study_on_gender_inequality_in_higher_education_in_Indian_context on April 25, 2019.

[7] Nath,S(2014).Higher Education and Women Participation in India. Journal of Business Management & Social Sciences Research(JBM&SSR),3(2),43-47 Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://www.borjournals.com/a/index.php/jbmssr/article/download/391/989&ved=2ahUKEwjxuNuuz77hAhV68HMBHeLeD0YQFjAFegQIBhAB&usg=AOvVaw3Sy6wip-9_tNW1Gdq4prOV&cshid=1554662309602 on April 22,2019.

[8] Sheikh, Y.A. (2017).Higher Education in India: Challenges and Opportunities. Journal of Education andPractice,8(1),39-42Retrievedfromhttps://www.google.com/url?sa=t&source=web&rct=j&url=https://files.eric.ed.gov/fulltext/EJ1131773.pdf&ved=2ahUKEwiY4Iv3wr7hAhWt8HMBHcTvD5wQFjAAegQIARAB&usg=AOvVaw2etedQVBucx0089wfB2f-X on April 25,2020.

[9] Shaukat, S. & Pell, A.W.9 (2015). Personal and Social Problems Faced by Women in Higher Education. FWU Journal of Social Sciences, winter 2015, 9(2): 101-109. Retrieved from https://www.semanticscholar.org/paper/Personal-and-Social-Problems-Faced-by-Women-in-Shaukat-Pell/fef9fe46cfd38134948f2600da2e2afdb62a719b on April 22, 2019

Web sources:

[10] A Statistical Analysis of Child Marriage in India, Based on Census 2011. (2017, June). Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=http://ncpcr.gov.in/showfile.php%3Flang%3D1%26level%3D1%26%26sublinkid%3D1213%26lid%3D1463&ved=2ahUKEwig19uphuzhAhVLvo8KHV1zBIwQFjAAegQIARAB&usg=AOvVaw1blvKsOPUeZ1YyIwPOAAA4&cshid=1556223171827 on April 25,2019.

[11] All India Survey on Higher Education (2016-17): Government of India, Ministry of Human Resource Development, Department of Higher Education, New Delhi.(2017) Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=http://aishe.nic.in/aishe/viewDocument.action%3FdocumentId%3D239&ved=2ahUKEwj07u_b5MXhAhUBbrwKHRMvBmAQFjABegQIBRAM&usg=AOvVaw0n9UnLtnvBaBaLb_pX5EWq on January 25,2019

[12] Agarwal.N(2014).Addressing gender inequality Indian homes ,archives. http://slis.simmons.edu/blogs/naresh/2014/03/08/international-womens-day-addressing-gender-inequality-in-indian-homes/ on April 25,2019.

[13] Census of india 2011. Retrieved from http://censusindia.gov.in/Census_And_You/gender_composition.a on April 10,2019

[14] Gohain, M.P. (2018, March 20).50% increase in sexual harassment cases on campus in 2017: Minister. Retrieved from https://www.google.com/amp/s/m.timesofindia.com/home/education/news/50-increase-in-sexual-

harassment-cases-on-campuses-in-2017-minister/amp_articleshow/63381826.cms on April 26,2019.

[15] Manjunathan,D et al(n.d.). Status_of higher_education_in_india_madras_university. Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://www.educationforallinindia.com/ . on April 7,2019.

[16] Singh, J.D.(2011). Higher Education in India – Issues, Challenges and Suggestions: "Higher Education", (pp.93-103). LAMBERT Academic Publishing, Germany. Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://www.researchgate.net/publication/282293148_Higher_Education_in_India__Issues_Challenges_and_Suggestions&ved=2ahUKEwj40eyw7cXhAhWKyYsBHf2GAN8QFjASegQIBxAB&usg=AOvVaw3DTifAVubc6eUc5a-NM5ro&cshid=1554912281720 on April 25,2019.

[17] Sharma, R.R. (2016, May 1). Gender disparity in Higher Education.Retrieved from https://www.dailyexcelsior.com/gender-disparity-in-higher-education/ on April 10,2019.

[18] Setlur, V.A.(2018,April 4). Sexual harassment in colleges- what you need to know. Retrieved from https://www.google.com/amp/s/yourstory.com/2018/04/sexual-harassment-colleges-need-know/amp on April 26, 2019.

[19] Sharma, A.(n.d.) What are the functions of the National Council For Women's Education?. Retrieved from http://www.preservearticles.com/2012030725538/what-are-the-functions-of-national-council-for-womens-education.html on April 11,2019.

[20] State of literacy2011. Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=http://censusindia.gov.in/2011-provresults/data_files/india/Final_PPT_2011_chapter6.pdf&ved=2ahUKEwiE77HJ_8XhAhUKhbwKHRHdBjUQFjAYegQIBBAC&usg=AOvVaw007E71NBjqghE-C6vBe80v on April 10,2019.

[21] The world Bank in India . Retyrieved from http://www.worldbank.org/en/country/india/overview on April 11,2019

[22]The Report of the University Education Commission: (1950). Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=http://www.educationforallinindia.com/1949%2520Report%2520of%2520the%2520University%2520Education%2520Commission.pdf&ved=2ahUKEwirqKCrgsvhAhXFr48KHWvFAA0QFjAMegQIBhAB&usg=AOvVaw14fKoAEKUxg23mCaYWw_1l on April 12,2019.

[23]The meaning of Forced Labour.(2014, March 10). Retrieved from https://www.ilo.org/global/topics/forced-labour/news/WCMS_237569/lang--en/index.htm on April 27, 2019.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs

[24]The Gazette of India. (2016, May 2). Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://www.ugc.ac.in/pdfnews/7203627_UGC_regulations-harassment.pdf&ved=2ahUKEwiM5vG49e3hAhV87HMBHXjIAysQFjAAegQICRAC&usg=AOvVaw3zjt7AiyHhgPHsTmqZ2lBT&cshid=1556288768519 on April 26,2019.

[25] Office of the Register General & Census Commissioner, India: Ministry of Home Affairs, Government of India.(n.d). Retrieved from http://censusindia.gov.in/2011-Common/CensusInfo.html on April 10,2019

[26] Orthodox principal of Indian society. Retrieved from https://respectwomen.co.in/orthodox-principles-of-indian-society/ on April 25,2019

[27] UNIVERSITY GRANT COMMISSION: "Consolidated list of All University". Pdf. (n.d.). Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://www.ugc.ac.in/oldpdf/consolidated%2520list%2520of%2520All%2520universities.pdf&ved=2ahUKEwjfvJCoxL7hAhW87nMBHcoDKoQFjATegQICBAB&usg=AOvVaw0KUwqo1McACZOk76bDwPgn6/4 on April 07,2019.

[28] What is forced labour, modern slavery and human trafficking.(n.d.) Retrieved from https://www.ilo.org/global/topics/forced-labour/definition/lang--en/index.htm on April27, 2019

[29] https://successfullady.in/problems-of-women-education-in-india/ Retrieved on April 26, 2019.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

JOURNAL OF INFORMATION AND COMPUTATIONAL SCIENCE

JOURNAL OF INFORMATION AND COMPUTATIONAL SCIENCE

ISSN: 1548-7741

submitjoics@gmail.com

CALL FOR PAPERS GUIDELINES INDEXING

EDITORIAL BOARD HOME AIM & SCOPE

CONTACT ARCHIVES PROPOSAL

VOL-11-ISSUE-11-2021

1. An Approach For Event Detection From News In Indian Languages

R.Priyanka, PG Scholar – Dept. of Information Technology, JNTUH College of Engineering, Kondagattu, Jagtial, Telangana India

Dr.S.Suresh Kumar, Assistant Professor and HOD – Department of Information Technology,Jawaharlal Nehru Technological University College of Engineering ,Jagital,Telangana,India

Page No: 1 – 10

DOI:10.12733.JICS.2021.V11I11.535569.36801

2. Rhetorical Roles for Labeling LegalJudgement Using BERT Model

G.Sailaja, PGScholar – Dept.OfInformation Technology,JNTUH College ofEngineering,Kondagattu,Jagtial,TelanganaIndia

Dr.S.Suresh Kumar, Assistant Professor and HOD – Department of Information Technology,Jawaharlal Nehru Technological University College of Engineering ,Jagital,Telangana,India

Page No: 11 – 21

A Test of Weak-form Efficiency of the Nifty Fifty Stock Index and its Predictability During the Pandemic

Dr. Madhuchhanda Lahiri

Assistant Professor, Department of Economics,
P.N. Das College, Palta, N-24 Parganas,
West Bengal, India.
e-mail id: macharyy20@gmail.com

Abstract

The study tests the weak-form efficiency of the Nifty Fifty Stock Price Index under the current pandemic situation. We use weekly data on closing prices of the nifty fifty index for the period January 2020 to December 2020 which was computed from the daily closing prices. Using the ACF and PACF coefficients, the corresponding correlograms and the ADF tests, we conclude that the price series is non-stationary. The first differenced series becomes stationary. Consequently, we find that the weekly return series is stationary, conforms to the normal distribution and also is weak-form efficient on the basis of the Runs test and the Kolmogorov -Smirnov test results. Thus we find that it was not possible for an investor to reap abnormal profits during the period of our study.

Keywords: *Stock market, nifty fifty index, share prices, weak-form efficiency*

1. Introduction

A stock market is considered to be weak-form efficient if current share prices fully reflect all information contained in their historical prices. This implies that no investor in stock market can devise a trading rule based solely on past share price patterns to earn abnormal returns. Stock market efficiency is an important parameter to examine the nature of financial system in any country.The study selects the S & P CNX Nifty which is an index of well-diversified 50 stocks index accounting for 21 sectors of the economy.

The basic objective of the study is to examine the behaviour of the Nifty Fifty stock Index after the introduction of the complete economic shutdown following the arrival and spread of the corona virus pandemic in India in early February 2020. We aim to study the stationarity of the original series, whether it conforms to the normal distribution and further how,to make the initially non-stationary series, a stationary one. Finally, we check the weak form efficiency of the stationary series.

2. Database and Methodology

Data for the Nifty Fifty Index comprising of 50 constituent companies were collected for our study period from 1st January 2020 to 31st December 2020. The daily closing prices of the Nifty Fifty index were collected from the NSE website for our study period of 1st January 2020 to 31st December 2020 (omitting those days when there was no trading of the stock or the stock exchange was closed for a particular day). We then found out the weekly closing prices of each stock by considering the closing price of the stocks traded on the last working day of the week (which is normally a Friday or the preceding day, if Friday is a holiday). Thereafter, the weekly price data has been transformed by taking natural log and then their first differences. This is nothing but the transformation of price data into weekly return data: $\ln (P_t/P_{t-1}) = \ln (P_t)-\ln (P_{t-1}) = r_t$; where r_t is the continuously compounded rate of change in the stock price, i.e., return for the week t; P_t is the price of the stock for the week t; P_{t-1} is the same for the preceding week and ln is the natural logarithm. The transformation of the price data has been justified by Granger and Morgenstern (1970) on the following basis: a) the distribution of prices is bounded from below at 0 but unbounded from above. The logarithmic transformation results in a distribution which is symmetrically unbounded and b) the transformed data is more stable and stationary in terms of mean and variance.

To illustrate the notion of random walk, we assume that the price of a stock at time t is equal to its price at time (t-1) plus a random shock u_t which is a white noise error term with 0 mean and variance σ^2.

$y_t = y_{t-1} + u_t$

Random Walk Model with or without drift is a non-stationary stochastic process. We also note that though y_t is non-stationary, its first difference is stationary $\Delta y_t = y_t - y_{t-1} = u_t$.

3. Analysis

The sequence plot of the weekly closing prices over our study has been plotted in Fig 1. It shows that there exists marked trend in the weekly nifty fifty index prices. Thus, the original time series appear to be non-stationary.

15/8/23

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs

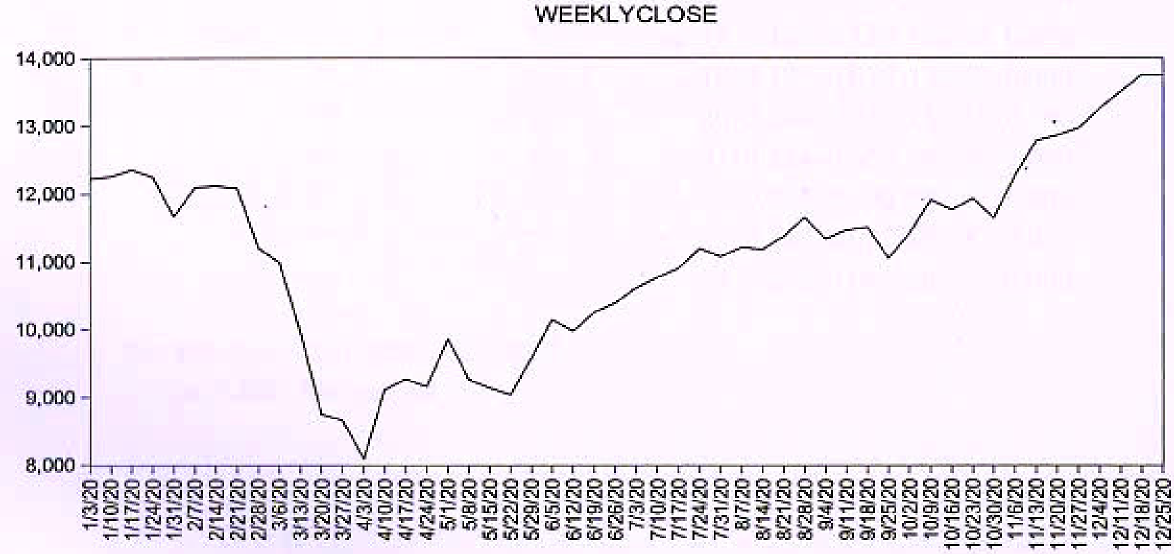


Figure 1. Sequence Plot of the Weekly Close Prices of Nifty Fifty Index over the Study Period

The corresponding correlograms of the ACF and PACF for the original series are plotted in Fig 2. The PACF is significant at lag 1 but thereafter, it becomes insignificant; however, the ACF coefficients are significant till lag 6 and shows gradual dampening effect. Thus, we can conclude that the original series is non-stationary.

Table 1. ACF and PACF Coefficients of Weekly Closing Prices of Nifty Fifty Index

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>. |*******</td><td>. |*******</td><td>1</td><td>0.914</td><td>0.914</td><td>46.007</td><td>0.000</td></tr>
<tr><td>. |******|</td><td>.*| . |</td><td>2</td><td>0.816</td><td>-0.117</td><td>83.438</td><td>0.000</td></tr>
<tr><td>. |***** |</td><td>.*| . |</td><td>3</td><td>0.706</td><td>-0.125</td><td>112.04</td><td>0.000</td></tr>
<tr><td>. |**** |</td><td>.*| . |</td><td>4</td><td>0.593</td><td>-0.081</td><td>132.59</td><td>0.000</td></tr>
<tr><td>. |**** |</td><td>. |*. |</td><td>5</td><td>0.505</td><td>0.099</td><td>147.81</td><td>0.000</td></tr>
<tr><td>. |*** |</td><td>.*| . |</td><td>6</td><td>0.398</td><td>-0.194</td><td>157.50</td><td>0.000</td></tr>
<tr><td>. |** |</td><td>. | . |</td><td>7</td><td>0.309</td><td>0.032</td><td>163.46</td><td>0.000</td></tr>
<tr><td>. |** |</td><td>.*| . |</td><td>8</td><td>0.219</td><td>-0.079</td><td>166.53</td><td>0.000</td></tr>
<tr><td>. |*. |</td><td>. | . |</td><td>9</td><td>0.143</td><td>0.034</td><td>167.87</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>10</td><td>0.063</td><td>-0.157</td><td>168.14</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>. |*. |</td><td>11</td><td>0.000</td><td>0.089</td><td>168.14</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>12</td><td>-0.043</td><td>0.006</td><td>168.27</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>13</td><td>-0.079</td><td>0.007</td><td>168.72</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>. | . |</td><td>14</td><td>-0.086</td><td>0.063</td><td>169.26</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>15</td><td>-0.109</td><td>-0.117</td><td>170.16</td><td>0.000</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>16</td><td>-0.136</td><td>-0.093</td><td>171.61</td><td>0.000</td></tr>
</table>

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

.*|. |	.|. |	17	-0.153	0.040	173.49	0.000
.*|. |	.*|. |	18	-0.186	-0.119	176.36	0.000
.*|. |	.|. |	19	-0.197	0.059	179.66	0.000
**|. |	.|. |	20	-0.208	-0.029	183.45	0.000
**|. |	.*|. |	21	-0.224	-0.068	187.99	0.000
**|. |	.|. |	22	-0.229	0.000	192.91	0.000
**|. |	.*|. |	23	-0.248	-0.087	198.87	0.000
**|. |	.|. |	24	-0.262	-0.010	205.74	0.000

Sample data: 1/03/2020 12/25/2020
Included observations: 52

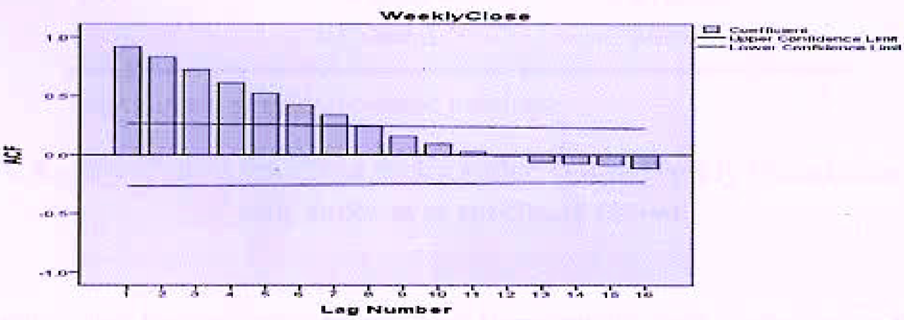


Figure 2. Correlogram of the ACF of Weekly Close Prices of Nifty Fifty Index over our Study Period

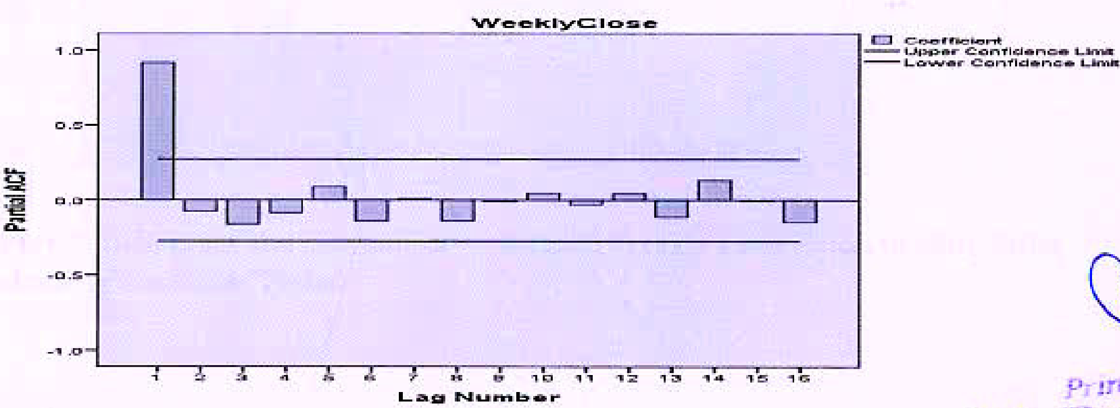


Figure 3. Correlogram of the PACF of Weekly Closing Prices of Nifty Fifty Index over our Study Period

The results of the Augmented Dicky-Fuller Test are shown in Figure 4. The ADF tests –with drift but without trend, and with drift as well as with trend - are conducted. Here the

Null hypothesis H_0 states that the Close Price series is non-stationary, i.e., it has a unit root. As the p-value of the test- statistic is greater than 0.05, (p= 0.9248), we accept the Null Hypothesis that the original series is non-stationary.

Null Hypothesis: WEEKLYCLOSE has a unit root
Exogenous: Constant, linear trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.390972	0.9027
Test critical values:	1% level	-3.565430	
	5% level	-2.919952	
	10% level	-2.597905	

*MacKinnon (1996) one-sided p-values.

Figure 4. Results of the Augmented Dicky Fuller Test of Weekly Close Prices of Nifty Fifty Index over our Study Period

Figure 5 and Figure 6 give the results of the normality tests conducted on the original close price series. One sample Kolmogorov-Smirnov Test shows that the distribution does not conform to the normal distribution.

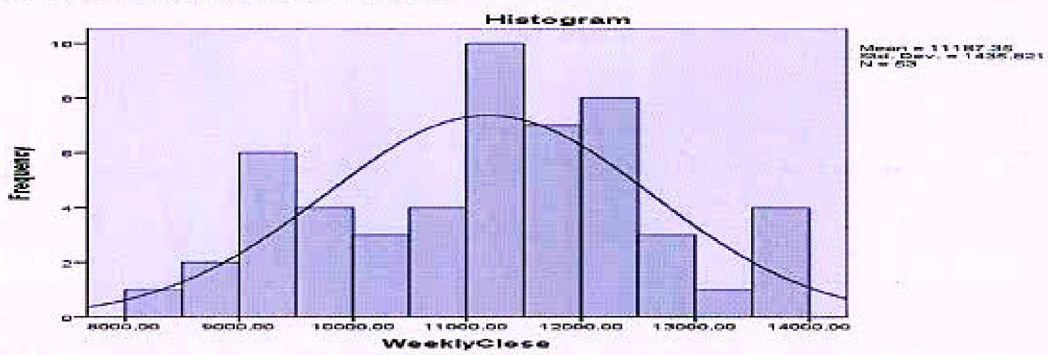


Figure 5. Histogram and Normal Curve fitted to Weekly Close Prices of Nifty Fifty Index over our Study Period

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

		WeeklyClosePrice
N		53
Normal Parameters[a,b]	Mean	11187.3528
	Std. Deviation	1435.82128
Most Extreme Differences	Absolute	.087
	Positive	.079
	Negative	-.087
Kolmogorov-Smirnov Z		.631
Asymp. Sig. (2-tailed)		.821

Figure 6. Result of One Sample Kolmogorov- Smirnov Test for Weekly Close Prices of Nifty Fifty Index over our Study Period

Following one of the basic assumptions of the random walk model that if the stock prices as well as the stock indices exhibit random walk then they will be non-stationary series, but their first differences (i.e., the stock returns or returns of the indices) will be stationary series. So, to make the original series stationary, we difference it once and plot the corresponding sequence plot of the first differenced series in Figure 7.

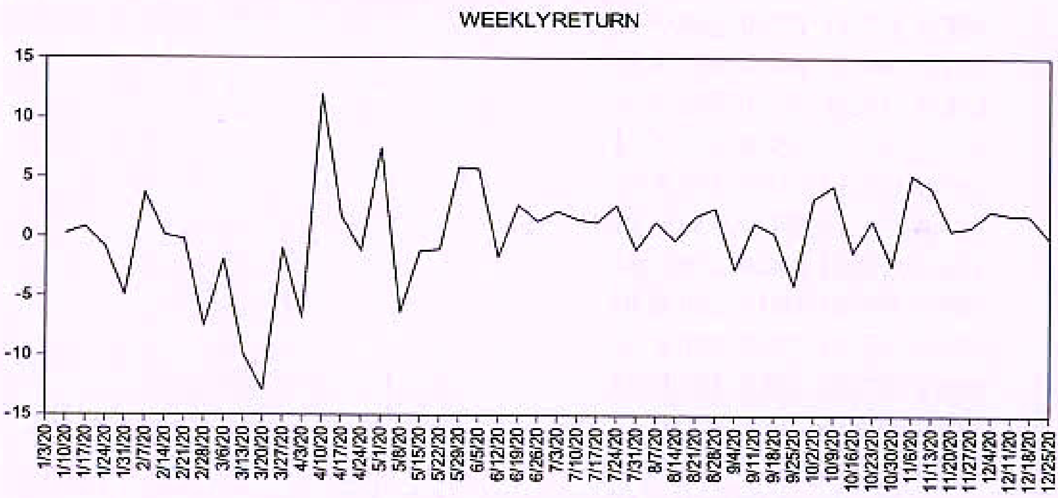


Figure 7. Sequence Plot of the Weekly Close Returns of Nifty Fifty Index over the Study Period

We find that the first- differenced series has become stationary without any trend component. The correlogram plot of the ACF and PACF of the first differenced series is shown in Figure 8 and Figure 9. The correlograms of the ACF and PACF corresponding to the weekly return series also lie well within the significant limits for all sixteen lags . The gradual damping effect observed of the significant ACF coefficients over the initial first few lags (till lag 7) along with the presence of a significant spike at lag 1 of the PACF coefficient also points towards the non-stationarity of the weekly close price series. However, in Figure 8 and Figure 9, we find that all the ACF and PACF coefficients are well within the lower and upper bounds till lag 16 unlike weekly close returns.

Table 2. ACF and PACF values of the Weekly Return of the Nifty Fifty Index

Sample: 1/03/2020 12/25/2020

Included observations: 51

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. |*. |	. |*. |	1	0.093	0.093	0.4653	0.495
. |*. |	. |*. |	2	0.189	0.182	2.4458	0.294
. |*. |	. | . |	3	0.101	0.072	3.0151	0.389
.*| . |	**| . |	4	-0.203	-0.262	5.3846	0.250
. |*. |	. |*. |	5	0.170	0.193	7.0855	0.214
.*| . |	.*| . |	6	-0.204	-0.181	9.5819	0.143
. |*. |	. |** |	7	0.189	0.250	11.775	0.108
. |*. |	. | . |	8	0.132	0.056	12.876	0.116
. | . |	. | . |	9	-0.022	-0.001	12.908	0.167
. | . |	**| . |	10	-0.027	-0.280	12.956	0.226
.*| . |	. | . |	11	-0.149	0.071	14.450	0.209
. | . |	. | . |	12	0.068	0.073	14.771	0.254
.*| . |	.*| . |	13	-0.154	-0.094	16.468	0.225
.*| . |	.*| . |	14	-0.077	-0.142	16.901	0.262
. | . |	. | . |	15	0.020	0.059	16.930	0.323
.*| . |	. | . |	16	-0.075	-0.057	17.366	0.362
. | . |	. | . |	17	-0.011	-0.042	17.374	0.429
.*| . |	. | . |	18	-0.092	0.040	18.065	0.451
. | . |	. |*. |	19	0.062	0.094	18.389	0.497
. | . |	. | . |	20	0.041	-0.051	18.536	0.552
.*| . |	.*| . |	21	-0.143	-0.163	20.372	0.498
. | . |	. |*. |	22	0.036	0.122	20.494	0.552
. | . |	. |*. |	23	0.035	0.120	20.614	0.605
. | . |	.*| . |	24	-0.025	-0.133	20.676	0.658

Figure 8. Correlogram of the PACF of the Weekly Returns of the Nifty Fifty Index over our Study Period

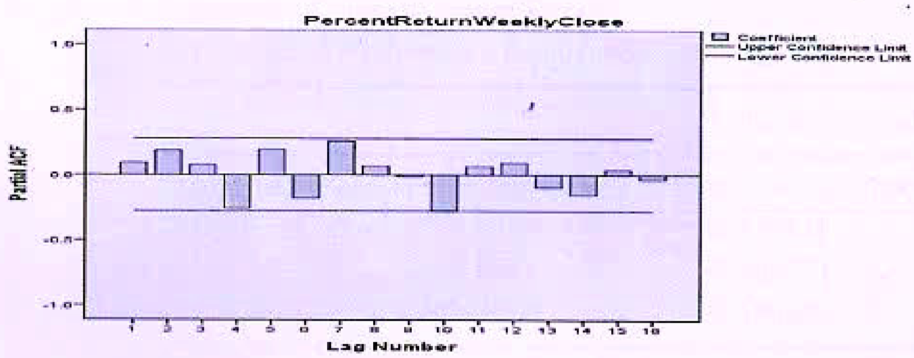
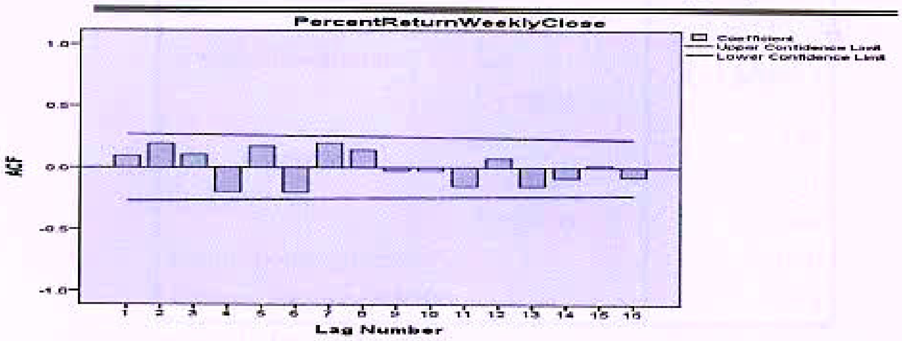


Figure 9. Correlogram of the PACF of the Weekly Return of the Nifty Fifty Index over our Study Period

The Augmented Dicky Fuller Test result for the first differenced series with and without trend is shown in Figure 10 and Figure 11. We find that the weekly return series is stationary over a constant mean and as the p values are both less than 0.05, so we cannot accept the null hypothesis of the presence of unit root and conclude that that the return series is stationary in both the forms of the Augmented Dicky Fuller Test.

Null Hypothesis: WEEKLYRETURNS has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)



		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.312241	0.0000
Test critical values:	1% level	-3.568308	
	5% level	-2.921175	
	10% level	-2.598551	

*MacKinnon (1996) one-sided p-values.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

Figure 10. Results of the ADF Test of the Weekly Return of the Nifty Fifty Index without Trend

Null Hypothesis: WEEKLYRETURNS has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.729412	0.0000
Test critical values:	1% level	-4.152511	
	5% level	-3.502373	
	10% level	-3.180699	

*MacKinnon (1996) one-sided p-values.

Figure 11. Results of the ADF Test with Trend of the Weekly Return of the Nifty Fifty Index over our Study Period

The one sample K-S test has a p- value less than 0.05, showing that the weekly return series does not conform to the normal distribution (Figure 12).

One-Sample Kolmogorov-Smirnov Test

		PercentReturnWeeklyClose
N		52
Normal Parameters[a,b]	Mean	.2579518
	Std. Deviation	4.16506411
Most Extreme Differences	Absolute	.144
	Positive	.107
	Negative	-.144
Kolmogorov-Smirnov Z		1.040
Asymp. Sig. (2-tailed)		.229

a. Test distribution is Normal.

b. Calculated from data.

Figure 12. Results of the Kolmogorov-Smirnov Test of the Weekly Return of the Nifty Fifty Index over our Study Period

We now perform the Run Test to check the weak-form efficiency of the weekly returns of the nifty fifty index over our study period and the result is presented in Figure 13. We find that as the p- value of the runs test is greater than 0.05 (p = .779), we cannot reject the null hypothesis and accept that the nifty fifty weekly returns index is weak - form efficient.

Runs Test

	WeeklyReturn
Test Value[a]	.81378
Cases < Test Value	26
Cases >= Test Value	26
Total Cases	52
Number of Runs	26
Z	-.280
Asymp. Sig. (2-tailed)	.779

a. Median

Figure 13. Result of Run Test of the Weekly Returns of Nifty Fifty Index over our Study Period

We repeat the stationary test for the corresponding series of weekly close prices of the Nifty Fifty index. . The non-stationarity of the weekly close prices has been shown in Fig 14 (the time plot showing a steep trend); the ADF test statistic value lying outside the critical region thereby accepting the null hypothesis that the series is non-stationary and has a unit root (in Fig 15). The one sample Kolmogorov – Smirnov test statistic lies within the acceptance region, so we accept the null hypothesis that the weekly close prices follow a normal distribution (Fig 17). Thus, we find that the weekly close prices behave identically with the daily close prices. We now plot the time plot of the weekly close returns which have been calculated as per the method described in the above section. We find that the weekly close return series has become stationary as shown in Fig 18. Thus the weekly returns has lesser volatility The ADF test results (Fig 21) also confirm the stationarity of the weekly return series with the test statistic not lying within the critical region (p = . Presenting a comparative tabulated result on the normality test comprising of the one sample Kolmogorov – Smirnov Test and Shapira- Wilk test for daily close prices and daily returns as well as weekly close prices and weekly returns in Fig 23, we find that the weekly return series follows normal distribution as both the p-values of the K-S test and Shapiro -wilk test are less than 0.05, so that we accept our null hypothesis of normality assumption.

4. Conclusion

The analysis of weak form efficiency of the nifty fifty index over our study period during the pandemic times from January 2020 to December 2020 is based on the weekly close price and the return series.The weekly close prices have been derived from the daily close prices by considering the last closing day of the week, which is generally a Friday or any other preceding day, if Friday is a holiday A detailed study of the weekly stock prices of the nifty fifty stock index showed that the series is non-stationary (the time plot showing a steep trend) and the ADF test statistic value lying outside the critical region thereby accepting the null hypothesis that the series is non-stationary and has a unit root. The one sample Kolmogorov – Smirnov test statistic lies within the acceptance region, so we accept the null hypothesis that the weekly close prices follow a normal distribution. However, we find that the weekly close prices have high short-term volatilities (compared to the daily close prices). Differencing the series once makes it stationary. The corresponding ACF and PACF coefficients for weekly return series are within the lower and upper bounds for all 16 lags (while a few significant coefficients are generally noticed for the daily return series). Finally we find that the weekly return series conforms to the normal distribution. As regards the efficiency of the nifty fifty index, we find that it is efficient in weak – form on the basis of the K-S test and run test. The severe economic lockdown measures undertaken in the pandemic year 2020 has failed to make the nifty fifty index inefficient in the weak-form. Consequently, on the basis of our weak – form test results we can conclude that it is not possible to reap abnormal profit during these abnormal times.

REFERENCES

1. E. Fama, "A Review of Theory and Empirical Work", Journal of Finance, Vol 26, (May 1970).

2. E. Fama, "Efficient Capital Markets II", Journal of Finance, Vol.46, No.5, (1991)

3. E. Fama, "Random walks in Stock Market Prices", Financial Analysts Journal, (January/February 1995).

4. E. Fama, "Market Efficiency, Long-Term Returns, and Behavioural Finance", Journal of Financial Economics (September 1998).

5. E. Fama, L. Fisher, M. Jensen, and R. Roll, The Adjustment of Stock Prices to New Information, International Economic Review, Vol.10, (February).

6. M. Obaidullah, "Stock Prices Adjustment to Half-Yearly Earnings Announcements – A Test of Market Efficiency", Chartered Accountant, Vol.38, 1990a.

7. M. Obaidullah, "Stock Market Efficiency in India: A Statistical Inquiry", Chartered Financial Analyst, July 1991 .

THE PROBLEMS AND ISSUES IN URBANIZATIO IN NORTH 24 PARGANAS DISTRICT, WEST BENGAL

Dr. Chandan Sarkar*
***Assistant Professor, Department of Geography,**
P.N. Das College, Palta, North 24 Parganas, West Bengal, India

ABSTRACT

Urban areas in North 24 Parganas district have been facing severe shortage of public services like urban water, sewerage facilities, congestions and transport facilities, educational facilities, health facilities etc. These urban problems have been occurred due to lack of investment and technology, insufficient financial resources as well as Planning, managerial capacities of local bodies and governments, inadequate level of infrastructure, impact of rapid urban growth etc. As a result numerous problems have emerged. In this paper an attempt has been made to review the different problems of ***urbanization urban development*** *like Urban Sprawl, Overcrowding, Housing, Slums,* Housing, Unemployment, water supply, sewerage and waste management, health, education etc.

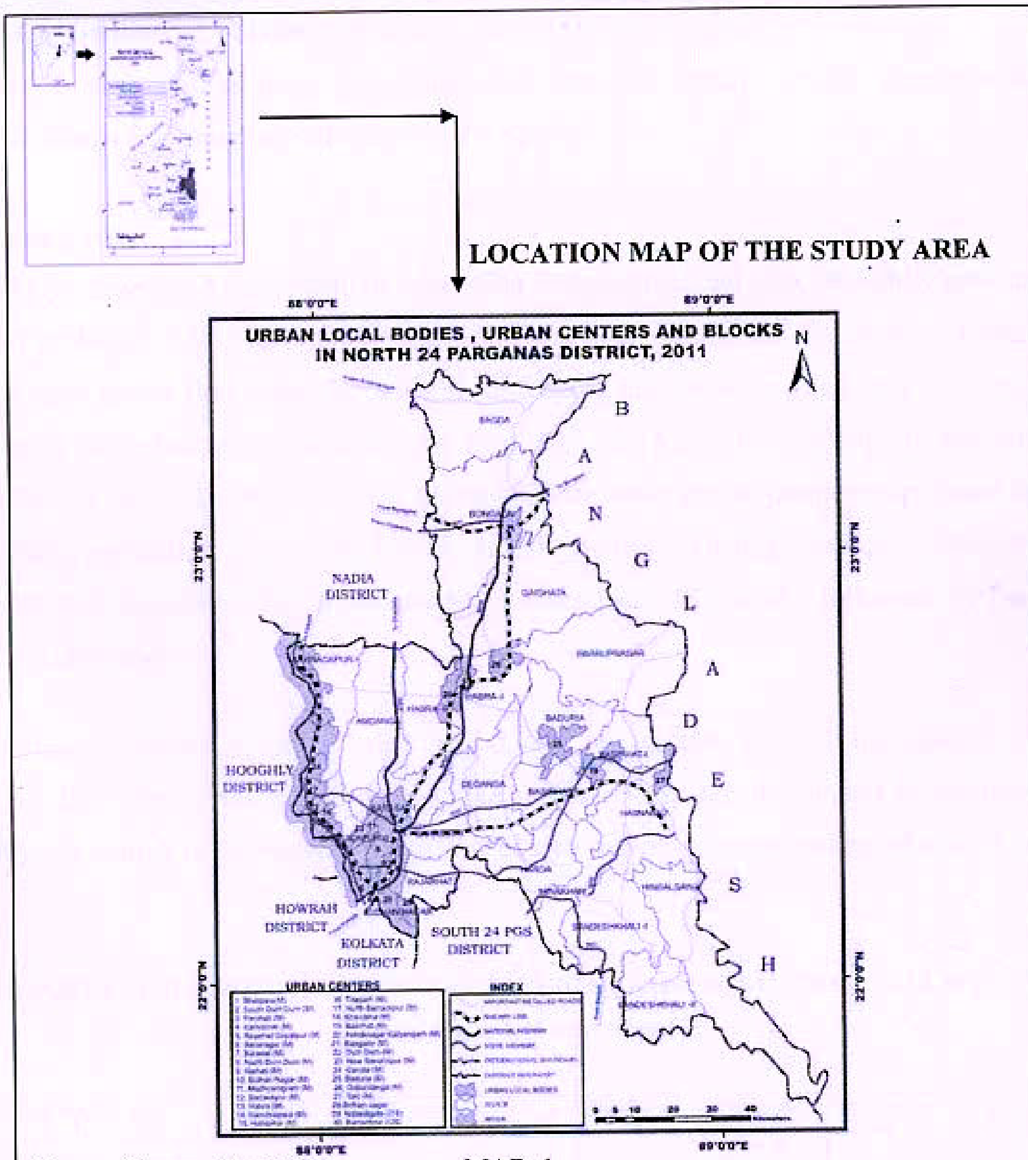
Keywords: Urban Problems, Urbanization, Urban Planning, Urban Development, Urban Growth

INTRODUCTION

The North 24 Parganas district is the highly urbanized (57.27 percent urban population, 2011) district in West Bengal. Due to high urbanization, haphazard and unplanned growth of towns, the rapid growth of urban population both due to natural growth and migration, has put heavy pressure on public amenities like housing, sanitation, transport, water, electricity, health, education and so on. So, the process of high pace of urbanization created wide gap between demand and supply of urban services and facing several urban crises and problems. The different problems of urbanization and urban development in North 24 Parganas district have been analysed in this paper.

STUDY AREA

The district North 24 Parganas forms the south-eastern part of the West Bengal and lies between 22°8' N to 23°16' N latitude and 88°18'E to 89°04' E longitude (Map-1). The district is bordered by the districts of Nadia in north, South 24 Parganas and Kolkata in the south, Howrah, Hooghly and Hooghly River in the west. Bangladesh forms the eastern international border of the North 24 Parganas district.



Source: Prepared by Author MAP- 1

DATA BASE AND METHODOLOGY

The study is based on primary and secondary sources of data. Secondary sources of data are mainly various Census Publications, Municipal Records of the District, Block Development Offices, Municipal Statistics of West Bengal and District Statistical Hand Book. The urbanization level and urban developmental level in each urban centres of the district have been taken and the simple statistical techniques like Correlation, Regression etc. have been applied for for analysing this paper.

OBJECTIVES

1. The main objective of this study is to analyze the different problems of urban growth, urbanization and urban development in North 24 Parganas district.
2. To study the urbanization processes created several urban problems.
3. To study the process of high pace of urbanization created wide gap between demand and supply of urban services and facing several urban problems.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

PROBLEMS OF URBANIZATION

The problems of urbanization have been discussed here of Urban Sprawl, Overcrowding, Housing, Unemployment, Slums and Squatter Settlement in the district.

1. URBAN SPRAWL

Urban sprawl of the towns and cities, both in population and geographical area, of rapidly growing cities, is the cause of urban problems. Massive immigration from rural areas is the basic problems of urbanization. The suburban areas have grown first along the major roads leading into the town. This type of growth is known as *ribbon settlement*. The urban areas increases have from 287.13sq k.m. to 451.43 sq k.m and urban population increase from 949,243 to 5,732,162 from 1951 to 2011. These urban sprawl problems are found high in an areas of high density population zone of Urban Local Bodies (ULBs), namely, Titagarh (35969.44), Baranagar(34440.03), Kamarhati(30128.74) and South Dum Dum (22456.35) followed by Panihati, Naihati, Khardaha and so on (Table -1).

The highly urbanized zones in the district are situated in western part of the district (under Kolkata Agglomeration). But other areas in the district, the urban centres are distributed in scattered pattern. So, population pressure is high in the western part of the district and also concentrations of sprawl are found along the area.

TABLE-1 POPULATION DENSITY AND NON-WORKING URBAN POPULATION OF ULBs IN NORTH 24 PARGANAS DISTRICT, 2011

Sl. No.	Urban Local Bodies (ULBs)	Civic Status and Class	Area in Sq.km.,2011	Population of 2011	Population Density Sq.km. in 2011	Non-Worker Percent
1	Ashokenagar Kalyangarh (M)	M--I	20.5	121592	5931.317	64.93
2	Baduria (M)	M--III	22.43	52493	2340.303	64.14
3	Bangaon (M)	M--I	14.27	108864	7628.872	63.59
4	Baranagar (M)	M--I	7.12	245213	34440.03	62.56
5	Barasat (M)	M--I	34.5	278435	8070.58	64.99
6	Barrackpore (M)	M--V	10.61	152783	14399.91	65.39
7	Barrackpur Cantonment (CB)	CB--III	3.68	17380	4722.826	67.14
8	Basirhat (M)	M--I	22.05	125254	5680.454	65.63
9	Bhatpara (M)+ OG(1)	M+OG(1)--I	34.69	386019	11127.67	68.7
10	Bidhannagar (M)	M--I	33.1	215514	6510.997	60.75
11	Dum Dum (M)	M--I	9.23	114786	12436.19	64.02
12	Garulia (M)	M--II	6.47	85336	13189.49	67.04
13	Gobardanga (M)	M--III	13.5	45377	3361,259	64.26
14	Habra (M)	M--I	21.8	147221	6753,257	63.41
15	Halisahar (M)	M--I	8.29	124939	15071.05	66.84
16	Kamarhati (M)	M--I	10.96	330211	30128.74	67.26
17	Kanchrapara (M)+ OG(3)	M+OG(3)--I	9.06	129576	14301.99	67.63
18	Khardah (M)	M--I	6.87	108496	15792.72	64.82
19	Madhyamgram (M)	M--I	21.5	196127	9122.186	64.46
20	Nabadiganta Industrial Township (ITS)	ITS- VI	1.76	1095	622.1591	57.63
21	Naihati (M)	M--I	11.55	217900	18865.8	67.33
22	New Barrackpore (M)	M--II	6.89	76846	11153.27	63.19
23	North Barrackpore (M)	M--I	12.61	132806	10531.8	67.68
24	North DumDum (M)	M--I	26.45	249142	9419.357	64.86
25	Panihati (M)	M--I	19.38	377347	19470.95	65.17
26	Rajarhat Gopalpur (M)	M--I	28	402844	14387.29	63.01
27	South Dum Dum (M)	M--I	17.96	403316	22456.35	62.38
28	Taki (M)	M--III	12.96	38263	2952.392	64.85
29	Titagarh (M)	M--I	3.24	116541	35969.44	68

Source: Compiled by Author from District Census handbook, 2011

2. OVERCROWDING

Overcrowding is the situation in which too many people live in too little space. Overcrowding is a logical consequence of over-population in urban areas. It is naturally expected that cities having a large size of population occupied in a small space must suffer overcrowding. The Overcrowding ULBs are Titagarh , Baranagar, Kamarhati, South Dum Dum, Panihati, Naihati and Khardaha. All the slum areas of towns are overcrowding zone. These ULBs and slum areas have tremendous population pressure on infrastructural facilities like housing, electricity, water, transport, employment etc.

3. HOUSING

Overcrowding leads to a chronic problem of shortage of houses in urban areas. The ULBs with high population density have to face huge short of land to accommodate huge population. The following Table-2 has been showing the distribution of households without owning land of Urban Local Bodies in North 24 Parganas district.

TABLE -2 DISTRIBUTION OF HOUSEHOLD WITHOUT OWNING LAND OF URBAN LOCAL BODIES IN NORTH 24 PARGANAS DISTRICT, 2010

Sl.No.	Urban Local Bodies (ULBs)	Percentage of Household Without Own Land
1	Titagarh (M)	63.66
2	Bidhannagar (M)	46.31
3	Garulia (M)	43.8
4	Kanchrapara (M)+ OG(3)	42.26
5	Bhatpara (M)+ OG(1)	41.87
6	Halisahar (M)	41.38
7	Rajarhat Gopalpur (M)	39.48
8	Kamarhati (M)	39.14
9	Baranagar (M)	35.21
10	South DumDum (M)	34.61
11	Dum Dum (M)	34.31
12	Barrackpore (M)	33.81
13	Naihati (M)	29.89
14	New Barrackpore (M)	28.6
15	Madhyamgram (M)	26.9
16	North Barrackpore (M)	26.15
17	Panihati (M)	24.96
18	North DumDum (M)	23.66
19	Bangaon (M)	23.25
20	Taki (M)	22.77
21	Barasat (M)	21.32
22	Khardah (M)	19.84
23	Habra (M)	17.7
24	Gobardanga (M)	15.21
25	Basirhat (M)	13.08
26	Ashokenagar Kalyangarh (M)	12.05
27	Baduria (M)	8.21

Source: District Human Development Report North 24 Parganas 2010

The 63.66 percent households in Titagarh municipality have not owned land. In Titagarh municipality, the population of working labour class are dominated and residential status is in temporary or is living in factory coolie lines. The other ULBs with high percentage of household without own land are found namely Bidhannagar (M) (46.31 percent), Garulia (M) (43.8 percent), Kanchrapara (M)+ OG(3) (42.26 percent), Bhatpara (M)+ OG(1)(41.87 percent), Halisahar (M)(41.38 percent) and so on (Table-2) . Actually the ULBs which are situated near the Kolkata and Hooghly industrial belt have to face the housing problems. The huge urban population have used to live in houses on encroached land, temporary settlements and rented houses. The housing problems are endemic in all the urban areas of the district.

4. UNEMPLOYMENT

The problem of unemployment is no less serious than the problem of housing and overcrowding mentioned above. The urban non-working population are also determining as the urban unemployment. The non-working populations are high in the ULBs of Bhatpara (M + OG), Titagarh (M), North Barrackpore (M), Kanchrapara (M + OG), Naihati (M), Kamarhati (M), Barrackpur Cantonment (CB), Garulia (M), Halisahar (M), Basirhat (M), Barrackpore (M) and Panihati (M) (Table-1). The unemployment features is the major problems in the urban areas of the district. Near about 64.87 percent urban population is non-working population in the district. So, urban unemployment in the district, have created several urban problems. This unemployment and poverty also leads to urban crimes. The urban poverty or urban poor population do not have to access the basic urban services. The majority of the urban poor largely get accommodated in slum and informal settlements.

5. SLUMS

The rapid urbanisation in conjunction with industrialisation has resulted in the growth of slums. The proliferation of slums occurs due to many factors such as, the shortage of developed land for housing, the high prices of land beyond the reach of urban poor, a large influx of rural migrants to the cities in search of jobs etc.

The following Table-3 has been showing the population features of slum area of ULBs in North 24 Parganas district.

TABLE-3 POPULATION FEATURES OF SLUM AREA OF ULBs IN NORTH 24 PARGANAS DISTRICT, 2011

Sl.No.	Name of the Town Having Slum	population Density in Sq. Km.	Slum Population	Percentage of Slum Population to Total Population
1	North Barrackpore (M)	10531.8	6972	5.25
2	Barrackpore (M)	14399.906	21265	13.92
3	Bhatpara (M + OG)	11127.674	74880	19.4
4	Rajarhat Gopalpur (M)	14387.286	82693	20.53
5	Panihati (M)	19470.949	90742	24.05
6	Dum Dum (M)	12436.186	28326	24.68
7	Kanchrapara (M + OG)	14301.987	33561	25.9
8	New Barrackpore (M)	11153.266	20394	26.54
9	Barasat (M)	8070.5797	73925	26.55
10	South DumDum (M)	22456.347	111066	27.54
11	Baduria (M)	2340.3032	14838	28.27
12	North DumDum (M)	9419.3573	70760	28.4
13	Garulia (M)	13189.49	29900	35.04
14	Bidhannagar (M)	6510.997	75900	35.22
15	Khardah (M)	15792.722	39650	36.55
16	Bongaon (M)	7628.8718	40258	36.98
17	Madhyamgram (M)	9122.186	75008	38.24
18	Ashokenagar Kalyangarh (M)	5931.3171	66049	54.32
19	Halisahar (M)	15071.049	84812	67.88
20	Gobardanga (M)	3361.2593	34740	76.56
21	Titagarh (M)	35969.444	112156	96.24

Source: Compiled by Author from District Census handbook, 2011

Figure- 1 has been showing the relationship between population density and slum population of ULBs in the district.

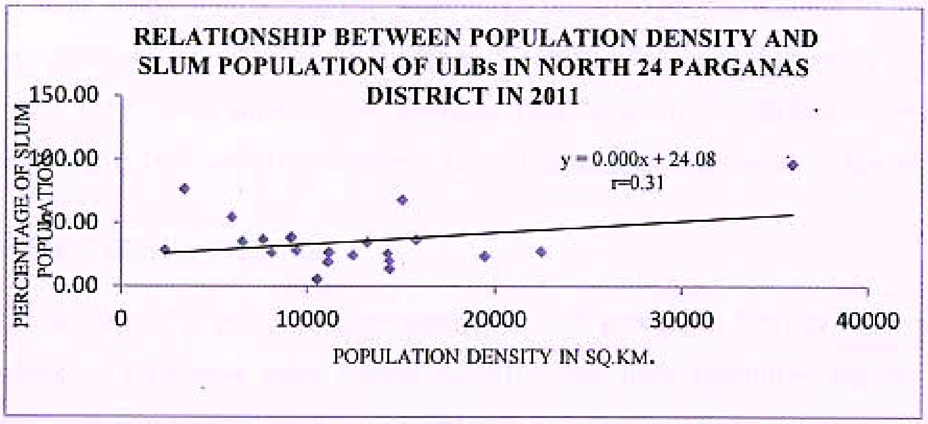


FIGURE -1

Slum is an integral part of urban areas. Slum population and urban poor are interlinked to each other. Unplanned and congested land use in slum areas leads to the problem of urban life with unhygienic environment. Three types of slums have been defined in Census of India 2011, namely Notified slum, Recognized slum and Identified slum.

(i)All notified areas in a town or city notified as 'Slum' by State, Union territories Administration or Local Government under any Act including a 'Slum Act' may be considered as Notified slums

(ii) All areas recognised as 'Slum' by State, Union territories Administration or Local Government, Housing and Slum Boards, which may have not been formally notified as slum under any act may be considered as Recognized slums.

(iii) A compact area of at least 300 populations or about 60-70 households of poorly built congested tenements, in unhygienic environment usually with inadequate infrastructure and lacking in proper sanitary and drinking water facilities. Such areas should be identified personally by the Charge Officer and also inspected by an officer nominated by Directorate of Census Operations. This fact must be duly recorded in the charge register. Such areas may be considered as Identified slums (Primary Census Abstract for Slum- 2011, pp. 4-5)

Out of 29 ULBs, 21 involving with slum population is showing in Table-3. The highest slum populated ULBs is Titagarh Municipality with 96.24 percent slum population. This area is congested with labour class population. Ashoknagar Kalyangarh Municipality, Halisahar Municipality and Gobardanga Municipality are involved with more than 50 percent slum population. The minimum slum population is in the North Barrackpore Municipality with 5.25 percent slum population and it is followed by Barrackpore, Bhatpara, and Rajarhat Gopalpur and so on in Table-3.

The relationship between urban population density and percentage of urban slum population is quite low positive relation. Where the value of r=0.31, means the high densely populated and high slum populated ULBs are corresponding to each other.

The rapid urbanization in conjunction with industrialisation has resulted in the growth of slums. The large number of slums occurs due to many factors, such as, the shortage of housing, the high prices of land beyond the reach of urban poor, a large influx of rural migrants to the cities in search of jobs etc. The following ULBs have to face the different problems of slum areas, namely, Titagarh (M), Gobardanga (M), Halisahar (M), Ashokenagar Kalyangarh (M), Madhyamgram (M), Bongaon (M), Khardah (M), Bidhannagar (M) and Garulia (M) respectively. Being highly slum areas these have to face different developmental problems (Table- 3).

6. PROBLEMS OF CENSUS TOWNS

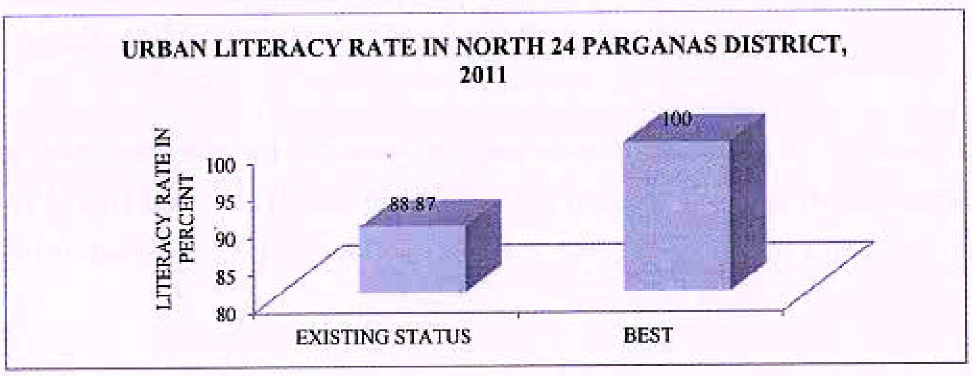
The Population of Census Towns is gradually increasing from 4.62 percent in 1951 to 12.74 percent in 2011. The maximum numbers of CTs have been formed in 2011, but their amenities depend upon the rural administration. So, the inadequate urban service and amenities are the major problems for CTs in the district. There is the administrative gap of these CTs area. When these CTs have transformed to ULBs, the problems will be solved.

PROBLEMS OF URBAN DEVELOPMENT

1. EDUCATION

The role of education is the major developmental indicator of any areas. Education is the most important input for empowering people with skills and knowledge and it is also giving them access to productive employment in future.

Many schools in the urban areas of North 24 Parganas district have shortage to basic amenities; they have no play grounds, drinking water, toilet facilities etc. Then it is necessary to disclose the problems related to urban education system in the district.



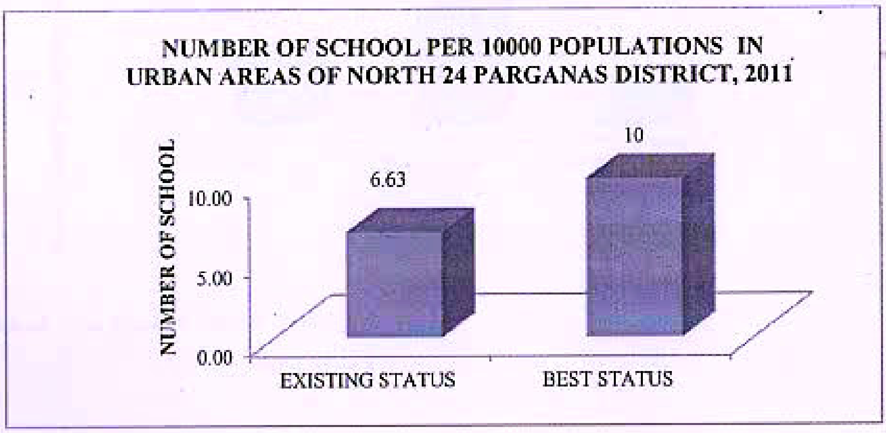
Source: Prepared by Author from District Census handbook, 2011

FIGURE- 2

The literacy rate is quite low in the urban areas of the district than the best level. The expected literacy rate is 100 percent but the present literacy rate is 88.87 percent.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

The number of schools per 10,000 population is the most important educational indicator. The number of 10 schools per 10 thousand population in the urban areas of the district is the standard status but the gap of it is 3.37(Fig-.3).



Source: Prepared by Author from District Census handbook, 2011

FIGURE- 3

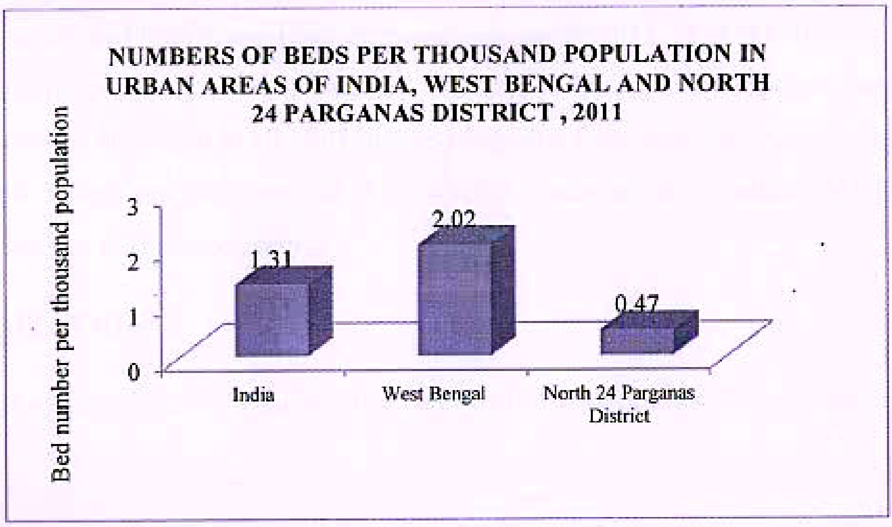
The number of technical and professional institutions in relation to population is very low at the urban areas of the district.

The educational infrastructure such as availability of various tools of education, educational furniture, playgrounds, library facilities reflect that the many government or private aided primary schools in the towns and cities of North 24 Parganas district are far behind the acceptable standards. The National Policy on Education (NPE,-1986, p.20) has provided the suggestion that every primary school will have to be upgraded to upper primary schools. However, the reality is quite different. But the urban areas in North 24 Parganas district have not these types of schools.

The ULBs having lower level educational status are found namely, in Naihati (M), Titagarh (M) and Garulia (M), Halisahar (M), North DumDum (M), Bhatpara (M + OG), Rajarhat Gopalpur (M), Kamarhati (M), Basirhat (M), Bidhannagar (M), Baranagar (M), South DumDum (M), Barrackpore (M) and Baduria (M) .

2. HEALTH

The scenario of health facilities in the urban areas of North 24 Parganas district is more or less same to the national level. In some cases, the condition is even quite low than the national level. All the major hospitals are situated at urban areas in the district. But the population pressure of the district is the basic problem against proper health services. All the hospitals or dispensaries are overcrowded in the sense that they exhibit high doctor-patient ratio. Although the number of hospitals has increased in urban areas of the district, but there has not been a corresponding increase in the other medical facilities required in the hospital like doctors, beds, medicines, medical staffs, nurses, dispensaries etc. Figure- 4 is showing the number of bed in India, West Bengal and North 24 Parganas district.



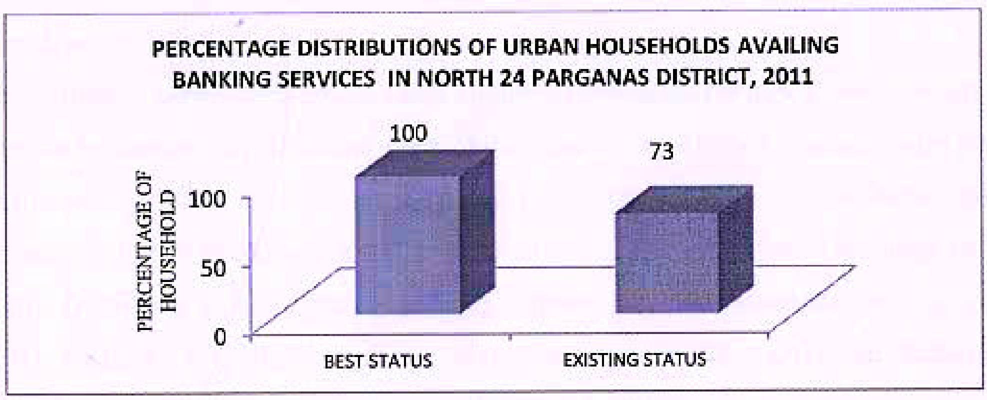
Source: Prepared by Author from District Census handbook, 2011

FIGURE- 4

Number of beds per 10,000 populations is very low in the district. Afore-said feature also indicates that the number of bed is quite low (0.47) than India (1.31) and West Bengal (2.20) (Fig-4). When the urban medical facilities access the people of rural areas, this ratio will more decrease and face to critical situation. The ULBs in the district namely Garulia, Halisahar, Kancharapara , North Dum Dum and taki, are no bed in health units. In the 22 ULBs, this ratio is below the state average. In some ULBs this ratio is also below from National level. So, the number of beds will have to increase to all the urban hospitals in the district for better health facilities.

3. BANKING

Banking facility is an important aspect for urban development. Because banking service determines the development of business, trade and commerce, industry and after all economic development. 73 percent urban household of the district have been availing the banking service by different of banks.



Source: Prepared by Researcher from District Census handbook, 2011

FIGURE- 5

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

The banking facilities are not totally available in maximum number of CTs in the district. One bank can serve properly the 10 thousand populations, but maximum numbers of ULBs have no proper banking services. Very low level banking services are found in 10 ULBs namely Rajarhat Gopalpur (M), Ashokenagar Kalyangarh (M), Kamarhati (M), North Dum Dum (M), Panihati (M), North Barrackpore (M), Naihati (M), Bhatpara (M + OG) , Garulia (M) and Halisahar (M) respectively .

4. URAN SANITATION

The urban sanitation system directly or indirectly related with the latrine facility, sewerage facilities and waste management.

4.1. Latrine Facilities

Proper sanitation system is a most important for urban development. The sanitation system is related with latrine facilities, drain facilities, proper sewerage system, and waste management. The Table-4 is showing latrine facilities of urban areas in North 24 Parganas district.

TABLE-4 LATRINE FACILITIES OF URBAN AREAS IN NORTH 24 PARGANAS DISTRICT, 2011

Piped Sewer System	Septic Tank	Pit Latrine	Others Latrine	No latrine within Premises Latrine
11 percent	54 percent	26 percent	3 percent	6 percent

Source: District Census Handbook, 2011

The Inadequate latrine facilities (less than10 percent) are in the ULBs namely Naihati (M), Bidhannagar (M), Basirhat (M), Habra (M), Gobardanga (M), Kanchrapara (M + OG), Barrackpur Cantonment (CB), Kamarhati (M), Barrackpore (M), Bongaon (M), North DumDum (M) and Baranagar (M). So, these urban areas need provisions of better latrine facilities for urban sanitation development.

4.2. Sewerage system

Proper drainage facilities in urban areas are the most important indicators for urban development. The percentage distribution of urban households by different types of drainage connectivity for waste water outlet in North 24 Parganas district is quite satisfactory. The existing status is 77 percent population of urban households which have drainage connectivity for waste water outlet in the district.. The inadequate drainage facilities are in the ULBs of North DumDum (M), Gobardanga (M), Habra (M), Bidhannagar (M),Taki(M), Barrackpur Cantonment (CB), Baduria (M), Bhatpara (M + OG), Kanchrapara (M + OG) and Nabadiganta Industrial Township (ITS) . More than 50 percent CTs have no drainage facilities.

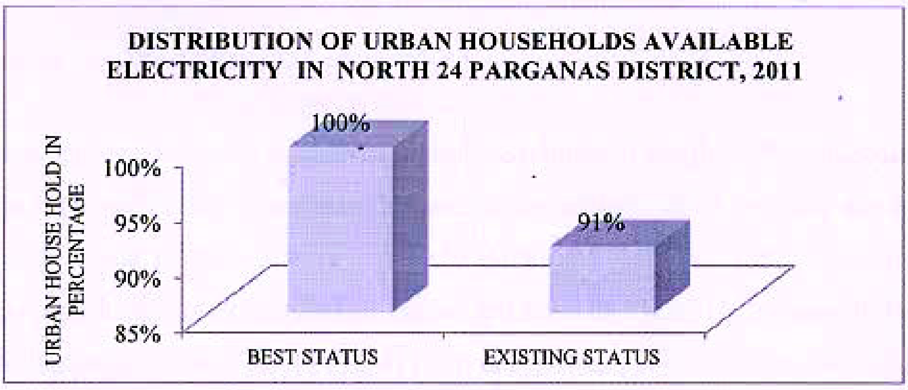
4.3. Solid waste materials

Urban areas are the most expected residential destination of people in developing nations. In rapidly urbanized areas in the district, solid waste management is one of the most essential services for maintaining the sound habitat for ensuring better standards of sanitation. In all the municipal areas, there are a gap between quantity of municipal solid waste generation per day and quantity of municipal solid waste collection per day. Near about 70 percent to 90 percent municipal solid waste materials have been collected for management. Rest of the solid waste materials are the major urban problems in the district.

5. URBAN POWER SUPPLY

Energy is an important indicator of urban infrastructure and development. Electricity is the main source of power supply. The electricity is used for various sectors like water supply, sewerage network, transportation, manufacturing, social infrastructure to enhance the quality of life, educational institutions, hospitals and daily household activities. So, without proper electricity facilities urban development is quite impossible.

Figure- 6 is showing the distribution of urban household's available electricity in North 24 Parganas district.



Source: Prepared by Researcher from District Census handbook, 2011

FIGURE- 6

Though, all the towns and cities of North 24 Parganas district have electricity facilities but about 9 percent in households have no electric connection.

6. TRANSPORT

North 24 Parganas district is highly urbanized district in West Bengal. The National and the State Highways as well as Railways are very important for transport system in the urban areas of the district.

All the CTs have to face the problems of un-metalled road conditions.

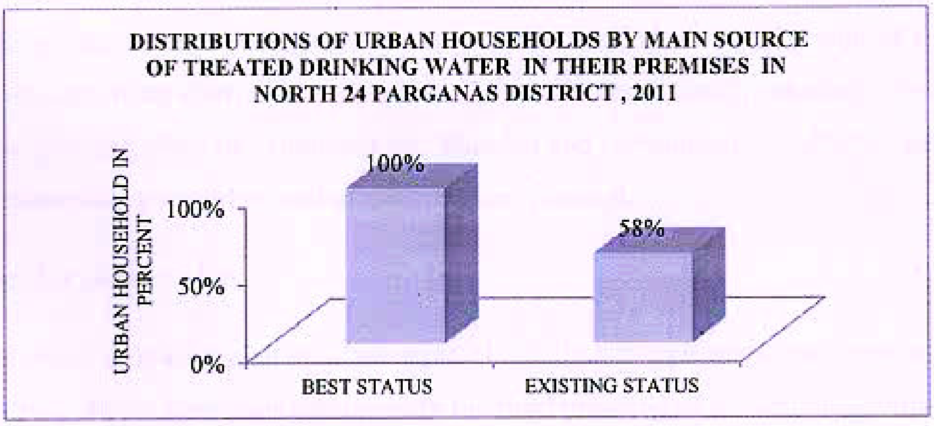
Maximum numbers of CTs and some ULBs are situated far distance from Kolkata and Barasat. Those ULBs and CTs do not get all the facilities easily like health, Education and other services.

Other transport problems are- **Rapid Vehicular Growth, Use of same road by all types transport, Parking Problem, Road encroachment and Traffic congestion.**

7. URBAN WATER SUPPLY

Proper water supply (drinking water and household's uses) is the major indicator for urban development. In India about 73 percent of the population living in Class-I Cities and 58 percent in Class-IV to VI category towns have access to piped water (GOI, Planning Commission, 2007).

Figure- 6 is showing the distribution of urban households by main sources of drinking water in their premises in North 24 Parganas district.



Source: Prepared by Author from District Census handbook, 2011

FIGURE- 7

The sources of treated drinking water 58 percent of urban households in North 24 Parganas district have accessed through tap water in their premises. More than 50 percent households of ULBs have not accessed the treated drinking water through piped in their premises namely Bashirhat, Baduria, Taki, Gobardanga, Bhatpara and Ashok Nagar- Kalayangarh. About 30 percent to 50 percent households of ULBs namely Bongaon, Halisahar, Titagarh, Habra, Bidhannagar, Naihati, South Dum Dum and Baranagar have not accessed the treated drinking water in their premises. So, availability of treated drinking water is the major problem in each municipal area of the district.

8. FINANCIAL PROBLEMS AND LACK OF PLANNING

Financial problem is the basic problem for development of urban areas of the district.

The Census Towns are totally controlled by rural administration. So, the financial supports for development in Census Towns depend upon the rural administration. So, all the Census Towns in North 24 Parganas district have to face the problems of finance and problems of planning for development.

9. PROBLEMS OF MASTER URBAN PLAN

Master plan of urban area is prepared with a long term vision (20-25 years) for development. The problems of master plan are - i) Non-availability of up-to-date and reliable data, ii) Lack of information on plan implementation strategies, iii) Lack information on financial strategy, iv) Absence of vital documents (Urban Development Act, land records), v) Lack of clarity on time frame for completion of plan, vi) Lack of public participation and follow-up the action on the feedback received from public, vii) Weak financial, managerial and technical capacity of implementing authorities, viii) Problems of land acquisition, ix) Violation of master plan and building regulations and ix) Absence of a separate monitoring mechanism for implementation.

SUGGESTIONS

1. Suggestions for Planning Urbanization

The urban Sprawl, population pressure and overcrowding due to rural migration are to be restricted for planned urbanization in urban areas of the district (especially Kolkata urban agglomeration zone in the district). At the same time, housing problems, urban crimes and so on are to be tackled. From this point of view to control the urban population pressure of the district, the urban areas have to increase the surroundings of suburban areas and to establish the new planning cities like Bidhannagar, Rajarhat and Nabadiganta. Lastly, the existing urban areas of the district for controlling over-urbanization need to be re-planned.

2. Suggestions for master plan

To minimize the problem of master plan in urban areas of the district some steps can be taken. The master plan of towns in the district, should keep coordination with the rural planning of the adjoining villages. There should be Coordination among the different level of planning authorities like- Panchayat level, Municipality level, District level, State level and National level planning. Public participation and Separate monitoring system for implementation of plan are very much important for master plan. Master plan should be prepared not only for the ULBs but also for the CTs and urban Outgrowths.

3. Recommendation for establishment of Nagar Panchayat

Urban Outgrowths and Census towns are the urban area under the rural administration. Proper developments of Outgrowths and Census Towns are held up due to the lack of proper planning and inadequate financial support by Gram Panchayat. Actually these both areas are in transition between rural and urban area. To govern this type of transitional area 74th Constitutional Amendment Act (1992) has recommended the Nagar Panchayat or Town Panchayat for the "transitional area". So, the Nagar Panchayats system has to be needed for urban development in North 24 Parganas district. The responsibilities of Town Panchayats are following civic services: providing basic Amenities: roads, street lights, water supply, public health, drainage; granting of building licenses; levying of taxes: property tax, vacant land tax, profession tax, water charges; issuing birth & death certificates; implementation of state and central schemes etc.

4. Recommendations for Development of Census Towns

All the Census Towns of the district are to be merged to nearby urban local bodies, for the permanent urban status and accession to the urban services, amenities, and infrastructure. In North 24 Parganas district, there are 78 CTs which are to be needed to merge to nearby ULBs or to establish as new ULB for developed urbanization in the district.

5. Government Incentives

Government should provide all types of amenities for rural migrants in urban areas of the district. This is essential for promoting human capital movement from rural to urban areas for urban economic development. Promoting the development of the rural economy surrounded by the towns is another effective way to correct rural-urban bias. Thus, rural migration gradually decreases.

6. Decentralization Power to Urban Local Bodies

In the12th Schedule of the Constitution of India, which includes urban planning, regulation of land use, construction of buildings, roads and bridges, water supply, and slum improvement that are performed by the Urban Local Bodies is cited but they have not implemented properly in urban areas in the district. The State Government is not decentralizing the power of ULBs for urban development. Therefore, functions of urban local bodies should be specified clearly and all the recommendations cited in the Twelfth Schedule should be transferred to urban local bodies along with funds and functions. Urban local bodies are becoming increasingly dependent on government grants for their developmental activities. For that, more responsibilities have to be handed over the ULBs, particularly decentralization of expenditure responsibilities.

7. Suggestions for Slum Development

The large number of slums occurs due to large influx of rural migrants to the cities in search of jobs. So, for slum areas, Government and ULBs have to take some steps for development like urban amenities, infrastructure, housing, job opportunities etc.

8. Creating Employment Opportunities

Government and ULBs should create employment opportunities for low-income communities through the promotion of small industries, handicrafts, micro-enterprises and public works. The employment opportunities also encourage and facilitate the participation of slum dwellers. So that it will aim at improving the quality of life and making cities free from the worst features of slums. Employment opportunities help to remove the urban poverty and urban crimes. Many small-scale agro-based industries and other industries need to be established for more job opportunities. Government should try to re-open the closed industries for economic development.

CONCLUSION

The overall discussion about the different problems of urban growth and urbanization in North 24 Parganas district brings out that the urban crises gradually increases. So, proper planned urbanization need to be done with effective and appropriate policies. For balanced urbanization proper land use is very much important in the study area. The urban population pressure will be made possible to reduce only after the development rural areas through proper infrastructure. Applications of all the above suggestions are most important to solve the urbanization problems in the district.

For proper development of urban areas in the district is very essential through different Government policies, social awareness, employment opportunities etc. The policy should be taken to develop more in urban services, infrastructure and urban economy in the district. Again, it is not desirable only to improve the infrastructure, facilities and services partially in the urban areas. The rural areas in the district also must be provided with better health, education, employment opportunity and other facilities. If rural and urban areas are developed equally, it will be sustainable development.

REFERENCES:

1. Agarwal, O.P. (2006): urban transport, India infrastructure report, 3i net work, oxford university press, 2006, new Delhi, p.114
2. Bhandari Laveesh (2006): Social Infrastructure Urban Health and Education, India Infrastructure Report, 2006, Oxford University Press, New Delhi, pp. 242, 233-253
3. District Census handbook (2011): Directorate Of Census Operations West Bengal, part XII-A and Part XII-B
4. District Human Development Report (2009): North 24 Parganas district, Development & Planning Department, Government of West Bengal.
5. District Statistical Hand book (2012-13): North 24 Parganas district. Bureau of Applied Economics & Statistics, Department of Statistics & Programme Implementation, Government of West Bengal.
6. GOI, Planning Commission HRD division (2012-2017): school education and literacy in the 12th plan Planning Commission of India, 2002, New Delhi, pp. 47-123
7. Islam, A (2011): Road Accident Analysis – a Case Study on Krishnanagar Urban Area, Journal of Inter academecia, 15(2),pp. 232-239.
8. Municipal statistics of West Bengal (2013-14): Bureau of applied economics & Statistics, Department of statistics & Programme implementation, Government of West Bengal.
9. National Policy on Education (1986): Programme on action in 1992, p. 20
10. Pacione, M. (2004): Urban Geography- A Global Perspective, 2nd edition, Routledge Publication, pp.577-586
11. Planning Commission India (2007): Eleventh Five Year Plan (2007-12), GOI, Volume-III, New Delhi.
12. Primary Census Abstract for Slum(2011): Office of the Registrar General & Census Commissioner, India, pp.4-5

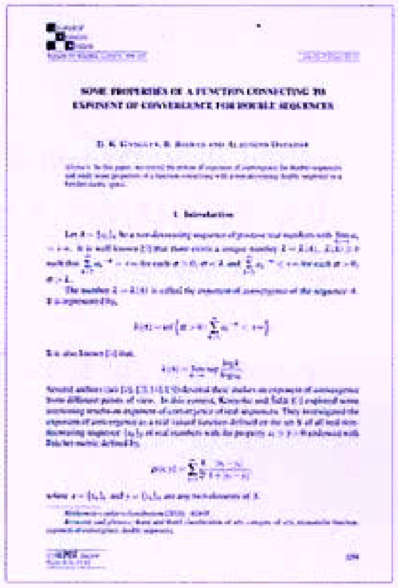
Journal of Classical Analysis

Open Access Journal

D. K. Ganguly, B. Biswas and Alauddin Dafadar

Some properties of a function connecting to exponent of convergence for double sequences

Recommend this article



DOI

- DOI name: dx.doi.org/10.7153/jca-10-11

Find all available articles from these authors

- D. K. Ganguly
- Bablu Biswas
- Alauddin Dafadar

Find more articles with the same subject classes

Primary

- 40A05

Find more articles with the same keywords

- Baire and Borel classification of sets
- category of sets
- measurable function
- exponent of convergence
- double sequences

Downloads

Abstract Abstract

Full article Full article

15/5/23

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

Journal of
Classical
Analysis
Volume 10, Number 2 (2017), 109–117

doi:10.7153/jca-10-11

SOME PROPERTIES OF A FUNCTION CONNECTING TO EXPONENT OF CONVERGENCE FOR DOUBLE SEQUENCES

D. K. GANGULY, B. BISWAS AND ALAUDDIN DAFADAR

Abstract. In this paper, we extend the notion of exponent of convergence for double sequences and study some properties of a function connecting with a non-decreasing double sequence in a Fréchet metric space.

1. Introduction

Let $A = \{a_k\}_k$ be a non-decreasing sequence of positive real numbers with $\lim\limits_{n\to\infty} a_n = +\infty$. It is well known [5] that there exists a unique number $\lambda = \lambda(A)$, $\lambda(A) \geqslant 0$ such that $\sum\limits_{k=1}^{\infty} a_k{}^{-\sigma} = +\infty$ for each $\sigma > 0$, $\sigma < \lambda$ and $\sum\limits_{k=1}^{\infty} a_k{}^{-\sigma} < +\infty$ for each $\sigma > 0$, $\sigma > \lambda$.

The number $\lambda = \lambda(A)$ is called the exponent of convergence of the sequence A. It is represented by,

$$\lambda(A) = \inf\left\{\sigma > 0 : \sum_{k=1}^{\infty} a_k{}^{-\sigma} < +\infty\right\}.$$

It is also known [4] that,

$$\lambda(A) = \limsup_{k\to\infty} \frac{\log k}{\log a_k}.$$

Several authors (see [1], [2], [4], [5]) devoted their studies on exponent of convergence from different points of view. In this context, Kostyrko and Šalát [5] explored some interesting results on exponent of convergence of real sequences. They investigated the exponent of convergence as a real valued function defined on the set S of all real non-decreasing sequence $\{x_k\}_k$ of real numbers with the property $x_1 > \gamma > 0$ endowed with Fréchet metric defined by,

$$\rho(x,y) = \sum_{i=1}^{\infty} \frac{1}{2^i} \frac{|x_i - y_i|}{1 + |x_i - y_i|},$$

where $x = \{x_k\}_k$ and $y = \{y_k\}_k$ are any two elements of S.

Mathematics subject classification (2010): 40A05.
Keywords and phrases: Baire and Borel classification of sets, category of sets, measurable function, exponent of convergence, double sequences.

Our aim in this paper is to extend the concept of exponent of convergence for double sequences of real numbers. During the last century, various problems connecting to the convergence and divergence of double sequences were treated by a number of mathematicians (see [6], [7], [8], [9], [10], [11]). Here first we give formulation for the exponent of convergence of double sequences which is analogous to the case of single sequences and then we prove some properties on exponent of convergence for the double sequences.

DEFINITION 1. [3] A double sequence $\{a_{mn}\}$ is said to be monotonically increasing (decreasing) if $a_{mn} \leqslant a_{pq}$ $(a_{mn} \geqslant a_{pq})$ holds if $(m,n) \leqslant (p,q) \iff m \leqslant p$ and $n \leqslant q$ under the partial ordering "$\leqslant$" on the set $\mathbb{N} \times \mathbb{N}$, where $\mathbb{N}$ is the set of positive integers.

DEFINITION 2. [3] A double sequence $\{a_{mn}\}$ of real numbers is said converge to the number ξ if for each $\varepsilon > 0$ there exists $(p,q) \in \mathbb{N} \times \mathbb{N}$ such that $|a_{mn} - \xi| < \varepsilon$ for $(m,n) \geqslant (p,q)$ and it is written as $\lim\limits_{m,n \to \infty} a_{mn} = \xi$.

DEFINITION 3. [3] A double series $\sum_{m,n=1}^{\infty} a_{mn}$ of real numbers is said to converge to ξ in Pringsheim sense if $\lim\limits_{m,n} s_{mn} = \xi$, where $s_{mn} = \sum\limits_{i=1}^{m} \sum_{j=1}^{n} a_{ij}$ is the partial sum of the double series.

Let S denotes the set of real non-decreasing double sequences $\{x_{mn}\}$ with $x_{11} \geqslant 1$, endowed with Fréchet metric defined by $\rho(x,y) = \sum\limits_{i,j} \frac{1}{2^{ij}} \frac{|x_{ij}-y_{ij}|}{1+|x_{ij}-y_{ij}|}$, where $x = \{x_{ij}\}_{i,j \geqslant 1}$, $y = \{y_{ij}\}_{i,j \geqslant 1}$ are points of S.

It is clear that, the set X of all real double sequences with Fréchet metric is a complete metric space. The convergence in this space is taken as co-ordinate wise convergence in Pringsheim sense.

In the following we establish some properties of the space (S,ρ).

THEOREM 1. *If S_1 is the collection of all non-decreasing sequences $\{x_{mn}\}$ of real numbers diverging to $+\infty$ with $x_{11} > 1$ then the space (S_1,ρ) is not a complete metric space.*

Proof. We consider a sequence $\{x^{(i)}\}_{i=1}^{\infty}$ of elements from S_1, where $x^{(i)} = \{a_{mn}^{(i)}\}_{m,n \geqslant 1}$ and

$$a_{mn}^{(i)} = \begin{cases} 1 + \frac{1}{i}; & \text{when} \quad (m,n) = (1,1) \\ m+n; & \text{otherwise.} \end{cases}$$

If $\{x^{(i)}\}_{i=1}^{\infty}$ converges to $x = \{a_{mn}\}$ then $a_{11} = 1$, since the space deals with the point wise convergence. Clearly, $x \notin S_1$ and consequently (S_1,ρ) is not complete. $\square$

THEOREM 2. *If T is the collection of all non-decreasing sequences $\{x_{mn}\}$ of real numbers diverging to $+\infty$ with $x_{11}=1$ then T is nowhere dense in S.*

Proof. Clearly, T is a closed subset of S. It is sufficient to prove that the complement of T in S is dense in S.

Let $a=\{a_{mn}\}\in S$. We consider an open ball $B(a,\delta)$, $\delta>0$ with centre at a and radius δ.

If $a_{11}>1$, then $a\in S-T$ and $(S-T)\cap B(a,\delta)\neq\phi$ and the result is proved.

Let $a_{11}=1$. Since, $\{a_{mn}\}$ is non-decreasing and diverges to $+\infty$ then there exists positive integers k and l for which $l+k$ is least such that $a_{kl}>1$.

For $\delta>0$, we can choose a real number $t_0>0$ such that $\frac{t_0}{1+t_0}<\delta$. Again, for any t with $0<t<t_0$, we have $\frac{t}{1+t}<\frac{t_0}{1+t_0}<\delta$.

We consider a sequence $b=\{b_{mn}\}$ such that

$$b_{mn}=\begin{cases}1+t_0; & \text{when} \quad (m,n)<(k,l)\\ a_{mn}; & \text{otherwise.}\end{cases}$$

Since, $b_{11}>1$ then $b\in S-T$ and

$$\begin{aligned}\rho(a,b) &= \sum_{i,j=1}^{\infty}\frac{1}{2^{ij}}\frac{|a_{ij}-b_{ij}|}{1+|a_{ij}-b_{ij}|}\\ &= \sum_{(i,j)<(k,l)}\frac{1}{2^{ij}}\frac{|a_{ij}-b_{ij}|}{1+|a_{ij}-b_{ij}|}\\ &< \frac{t_0}{1+t_0}<\delta.\end{aligned}$$

Then, clearly $b\in(S-T)\cap B(a,\delta)$ and thus $S-T$ is dense in S. Therefore, T is nowhere dense in S. □

THEOREM 3. *The set (S,ρ) is a complete metric space and has the cardinality of continuum.*

Proof. To prove the theorem we show that the set S is a perfect subset in (X,ρ).

Let $\{x^{(i)}\}_{i\geqslant 1}$ be any sequence in S, where $x^{(i)}=\{x^{(i)}_{mn}\}_{m,n\geqslant 1}$ and $i=1,2,3,\ldots$

If $i\to\infty$, $x^{(i)}\to x=\{x_{mn}\}$ (say) and then, $x\in S$. It readily follows from the pointwise convergence in Pringheim sense for double sequence with respect to the Fréchet metric. Hence, S is closed.

We now show that the set S is dense in it self.

We take any point $x=\{x_{mn}\}\in S$. We have to find a sequence $\{x^{(i)}\}_{i\geqslant 1}$ from S such that $\lim_{i\to\infty}x^{(i)}=x$.

We construct the sequence $\{x^{(i)}\}_{i\geqslant 1}$ as follows:

$${x_{mn}}^{(i)}=\begin{cases}x_{mn}, & \text{for} \quad (m,n)\leqslant(i,i)\\ x_{mn}+1, & \text{otherwise.}\end{cases}$$

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

It is clear that $\lim_{i\to\infty} x^{(i)} = x$.

Hence, S is a perfect set in (X,ρ) and thus, S has the cardinality of continuum and consequently the space (S,ρ) is complete having cardinality of continuum. □

2. Exponent of convergence

Let $A = \{a_{mn}\}$, $a_{mn} \geqslant 1$ be a non-decreasing double sequence of real numbers. If $\sigma < \tau$ then the convergence of the series $\sum_{m,n=1}^{\infty} a_{mn}^{-\sigma}$ implies the convergence of the series $\sum_{m,n=1}^{\infty} a_{mn}^{-\tau}$. This simple observation leads us to define the exponent of convergence $\lambda = \lambda(A)$ of the double sequence A as follows:

$$\lambda(A) = \inf\left\{\sigma > 0 : \sum_{m,n=1}^{\infty} a_{mn}^{-\sigma} < \infty\right\}.$$

Then, λ is a function from S to $[0,\infty]$.

The following result gives an alternative form for calculation of the exponent of convergence of double sequences under certain assumptions and it can be verified easily.

RESULT 1. If $\{a_{mn}\}$, $a_{mn} > 1$ be a non-decreasing double sequence of real numbers, then the exponent of convergence of the double sequence $\{a_{mn}\}$ is equal to the number $\limsup_{m,n\to\infty} \frac{\log mn}{\log a_{mn}}$.

We now examine some topological properties of the function λ.

THEOREM 4. *For any non-negative real number t there exists some $x \in S$ such that $\lambda(x) = t$.*

Proof. *Case* 1. Let $t = 0$. We choose $x = \{x_{mn}\}$, where $x_{mn} = (mn)^{mn}$. Then,

$$\lambda(x) = \limsup_{m,n\to\infty} \frac{\log(mn)}{\log(mn)^{mn}} = \limsup_{m,n\to\infty} \frac{1}{mn} = 0.$$

Case 2. Let $t > 0$. Then, there exists a positive integer k such that $((m+k)(n+k))^{\frac{1}{t}} > 1$ for positive integers m and n. Here, we choose $x = \{x_{mn}\}$, where $x_{mn} = ((m+k)(n+k))^{\frac{1}{t}}$. Then,

$$\begin{aligned}\lambda(x) &= \limsup_{m,n\to\infty} \frac{\log(mn)}{\log((m+k)(n+k))^{\frac{1}{t}}} \\ &= t.\limsup_{m,n\to\infty} \frac{\log mn}{\log(m+k)(n+k)} \\ &= t. \quad \square\end{aligned}$$

COROLLARY 1. *The function $\lambda : S \to [0,\infty]$ does not belong to Baire class one.*

Proof. It is well known that, the set of points of discontinuity of a function belonging to the Baire class one is a set of first category ([12]). As the function λ is totally discontinuous on S, it can not belong to Baire class one. $\square$

THEOREM 5. *For each real number d, the sets $A^d = \left\{x \in S; \lambda(x) < d\right\}$ and $A_d = \left\{x \in S; \lambda(x) > d\right\}$ belong to the third additive Borel class.*

Proof. First we investigate for the set A^d.

If $d \leqslant 0$, then clearly $A^d = \phi$ (null set) and the above result is true.

Let $d > 0$. Then, we have $A^d = \left\{x \in S : \lambda(x) < d\right\}$. So, there exists σ with $0 < \sigma < d$ for which

$$A^d = \left\{x \in S : \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{mn}^{-\sigma} < \infty\right\} = \bigcup_{k=k_0}^{\infty}\left\{x : \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{mn}^{-(d-\frac{1}{k})} < \infty\right\},$$

where k_0 is the smallest positive integer for which $\sigma = d - \frac{1}{k} > 0$, for all $k \geqslant k_0$.

We set, $H_k = \left\{x \in S : \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{mn}^{-\sigma} < +\infty\right\}$, where $\sigma = d - \frac{1}{k}$, $(k = k_0,\ k_0+1,\ k_0+2,\ldots)$.

Since, $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij}^{-\sigma} < +\infty$, for $\sigma = d - \frac{1}{k}$, $(k = k_0,\ k_0+1,\ k_0+2,\ldots)$, then by Stolz's theorem for convergence of double series of real terms, for each positive integer l, there exists $(m,n) \in \mathbb{N} \times \mathbb{N}$ such that

$$\sum_{j=n+1}^{\infty}\sum_{i=m+1}^{\infty} a_{ij}^{-\sigma} \leqslant \frac{1}{l}, \text{ for } l = 1,2,3,\ldots.$$

Then,

$$H_k = \bigcap_{l=1}^{\infty}\bigcup_{m=1}^{\infty}\bigcup_{n=1}^{\infty}\bigcap_{p=1}^{\infty}\left\{x \in S : \sum_{j=n+1}^{n+p}\sum_{i=m+1}^{m+p} a_{ij}^{-\sigma} \leqslant \frac{1}{l}\right\} = \bigcap_{l=1}^{\infty}\bigcup_{m=1}^{\infty}\bigcup_{n=1}^{\infty}\bigcap_{p=1}^{\infty} H_{klmnp},$$

where $H_{klmnp} = \left\{x \in S : \sum_{j=n+1}^{n+p}\sum_{i=m+1}^{m+p} a_{ij}^{-\sigma} \leqslant \frac{1}{l}\right\}$.

For fixed k,l,m,n,p, we prove that H_{klmnp} is a closed set.

This follows immediately from point-wise convergence of double sequence in Pringheim sense.

We take a sequence $\{a^{(s)}\}_{s=1}^{\infty}$ in H_{klmnp}, where $a^{(s)} = \{a_{ij}^{(s)}\}_{i,j\geqslant 1}$, $s = 1,2,\ldots$ and $\lim_{s\to\infty}(a_{ij}{}^{(s)}) = a$, where $a = \{a_{ij}\}_{i,j\geqslant 1}$.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

Then, $\lim_{s\to\infty}(a_{ij}{}^{(s)})^{-\sigma} = a_{ij}{}^{-\sigma}$, for all $(i,j) \in \mathbb{N}\times\mathbb{N}$.

Hence, $a \in H_{klmnp}$. Consequently, each of the sets H_{klmnp} is closed.

So, it follows that $A^d = \left\{x \in S : \lambda(x) < d\right\} = \bigcap_{l=1}^{\infty}\bigcup_{m=1}^{\infty}\bigcup_{n=1}^{\infty}\bigcap_{p=1}^{\infty} H_{klmnp}$ is an $F_{\sigma\delta\sigma}$ set in S and therefore, A^d belongs to the third additive Borel class.

We now investigate the set A_d.

If $d < 0$, then $A_d = S$ and the result is proved.

Let $d \geqslant 0$, then there exists a positive integer k such that

$$A_d = \left\{x \in S : \lambda(x) > d\right\} = \bigcup_{k=1}^{\infty}\left\{x \in S : \sum_{j=1}^{n}\sum_{i=1}^{m} a_{ij}^{-(d+\frac{1}{k})} = +\infty\right\}.$$

We set, $L_k = \left\{x \in S : \sum_{j=1}^{n}\sum_{i=1}^{m} a_{ij}^{-\sigma} = +\infty\right\}$, where $\sigma = d + \frac{1}{k}, k = 1, 2, \ldots$.

Thus, for each $p \in \mathbb{N}$ there exists $q \in \mathbb{N}$ such that $\sum_{j=1}^{n+q}\sum_{i=1}^{m+q} a_{ij} \geqslant p$, for all $(m,n) \in \mathbb{N}\times\mathbb{N}$.

We can express the set L_k as

$$L_k = \bigcap_{p=1}^{\infty}\bigcup_{q=1}^{\infty}\bigcap_{m=1}^{\infty}\bigcap_{n=1}^{\infty}\left\{x \in S : \sum_{j=1}^{n+q}\sum_{i=1}^{m+q} a_{ij}^{-\sigma} \geqslant p\right\} = \bigcap_{p=1}^{\infty}\bigcup_{q=1}^{\infty}\bigcap_{m=1}^{\infty}\bigcap_{n=1}^{\infty} L_{kpqmn},$$

where $L_{kpqmn} = \left\{x \in S : \sum_{j=1}^{n+q}\sum_{i=1}^{m+q} a_{ij}^{-\sigma} \geqslant p\right\}$.

Analogously as in the foregoing part of the proof, we can verify that each of the sets L_{kpqmn} is closed.

Hence, the set $A_d = \left\{x \in S : \lambda(x) > d\right\} = \bigcup_{k=1}^{\infty}\bigcap_{p=1}^{\infty}\bigcup_{q=1}^{\infty}\bigcap_{m=1}^{\infty}\bigcap_{n=1}^{\infty} L_{kpqmn}$ is an $F_{\sigma\delta\sigma}$ set and therefore A_d belongs to the third additive Borel class. □

COROLLARY 2. *The function $\lambda : S \to [0,\infty]$ is a measurable function.*

THEOREM 6. *For every real number d, each set $A^d = \left\{x \in S : \lambda(x) < d\right\}$ is of first category in S.*

Proof. By virtue of the above theorem, we can write

$$A^d = \bigcup_{k=k_0}^{\infty}\bigcap_{l=1}^{\infty}\bigcup_{m=1}^{\infty}\bigcup_{n=1}^{\infty}\bigcap_{p=1}^{\infty} H_{klmnp} = \bigcup_{k=k_0}^{\infty}\bigcap_{l=1}^{\infty} H_{kl},$$

where $H_{kl} = \bigcup_{m=1}^{\infty}\bigcup_{n=1}^{\infty}\bigcap_{p=1}^{\infty}\left\{x : x \in S : \sum_{j=n+1}^{n+p}\sum_{i=m+1}^{m+p} a_{ij}^{-\sigma} \leqslant \frac{1}{l}\right\}$.

We first prove that, each of the sets H_{kl} is of first category in S. For this, it is sufficient to show that, H_{kl} is an F_σ set whose complement is dense in S.

Principal
P.N. Das College
Palta, Bengal Enamel, N. 24 Pgs.

Let $z = \{z_{mn}\}_{m,n \geqslant 1} \in S$. For given $\varepsilon > 0$, let r be smallest positive integer such that $\sum\limits_{i=r+1}^{\infty} 2^{-i} < \varepsilon$. Take $y = \{(mn)^{\alpha}\}_{m,n \geqslant 1}$ where $\alpha = \frac{1}{\sigma}$.

We now construct a sequence $x = \{x_{mn}\}_{m,n \geqslant 1} \in S$ as follows:

Let $x_{mn} = z_{mn}$ for $m = 1, 2, \ldots, r$; $n = 1, 2, \ldots, r$.

If $x_{rr} \leqslant (1+r)^{\alpha}$, then put $x_{mn} = (mn)^{\alpha}$ for all (m,n) when $(m,n) \nleq (r,r)$.

If $x_{rr} > (1+r)^{\alpha}$, then put $x_{mn} = x_{rr}$ for $m = r+1, r+2, \ldots, s-1$; $n = r+1, r+2, \ldots, s-1$, where s is the smallest positive integer for which $s^{\alpha} \geqslant x_{rr}$ and $x_{mn} = (mn)^{\alpha}$ for all (m,n) when $(m,n) \nleq (s-1, s-1)$.

It is clear that $\rho(x,z) < \varepsilon$.

Again, there is a positive integer t for which

$$x_{mn} = (mn)^{\alpha} \text{ for } m = t, t+1, \ldots;\ n = t, t+1, \ldots$$

Then, there exist $q \in \mathbb{N}$, such that for all $(m,n) \in \mathbb{N} \times \mathbb{N}$ with $m \geqslant t$ and $n \geqslant t$ we have

$$\begin{aligned}\sum_{j=t}^{n+q} \sum_{i=t}^{m+q} x_{ij}^{-\sigma} &= \sum_{j=t}^{n+q} \sum_{i=t}^{m+q} (mn)^{-\sigma\alpha} \\ &= \sum_{j=t}^{n+q} \sum_{i=t}^{m+q} (mn)^{-1}.\end{aligned}$$

Since, the series $\sum\limits_{m=1}^{\infty} \sum\limits_{n=1}^{\infty} (mn)^{-1}$ is divergent, then x belong to the complement of H_{kl}. Therefore, the complement of H_{kl} is dense in S.

Also each of the sets H_{klmnp} is closed. Hence, $H_{kl} = \bigcup\limits_{m=1}^{\infty} \bigcup\limits_{n=1}^{\infty} \bigcap\limits_{p=1}^{\infty} H_{klmnp}$ is an F_{σ} set. Thus, each of the sets H_{kl} is of first category in S and consequently $A^d = \bigcup\limits_{k=k_0}^{\infty} \bigcap\limits_{l=1}^{\infty} H_{kl}$ is of first category. □

THEOREM 7. *The set* $\left\{x \in S : \lambda(x) = \infty\right\}$ *is residual in* S.

Proof. By Theorem 6, the set $\left\{x \in S : \lambda(x) < \infty\right\} = \bigcup\limits_{n=1}^{\infty} \left\{x \in S : \lambda(x) < n\right\}$ is of first category in S and also the space S is complete. Hence, the set $\{x \in S : \lambda(x) = \infty\}$ is residual in S. □

THEOREM 8. *For* $t > 0$, *the set* $A = \left\{x \in S : \lambda(x) = t\right\}$ *is dense in* S.

Proof. Let $y = \{y_{mn}\}_{m,n \geqslant 1}$ be any point in S. Let us choose $\varepsilon > 0$ and $(k,l) \in \mathbb{N} \times \mathbb{N}$ such that $\sum\limits_{i=k}^{\infty} \sum\limits_{j=l}^{\infty} \frac{1}{2^{ij}} < \varepsilon$.

We choose, $(u,v) \in \mathbb{N} \times \mathbb{N}$ such that for every $(m,n) \in \mathbb{N} \times \mathbb{N}$ with $(m,n) > (k,l)$ we have $\{(m+u)(n+v)\}^{\frac{1}{t}} \geqslant \max\{y_{mn} : (m,n) \leqslant (k,l)\}$.

Define a sequence $z = \{z_{mn}\}_{m,n\geqslant 1}$ such that

$$z_{mn} = \begin{cases} y_{mn}; & \text{for} \quad (m,n) \leqslant (k,l), \\ [(m+u)(n+v)]^{\frac{1}{t}}; & \text{for all} \quad (m,n) \quad \text{where} \quad (m,n) \nleq (i,i). \end{cases}$$

Then, clearly $z \in B(y,\varepsilon)$ and

$$\begin{aligned} \lambda(z) &= \limsup_{m,n} \frac{\log mn}{\log z_{mn}} \\ &= \limsup_{m,n} \frac{\log mn}{\log\{(m+u)(n+v)\}^{\frac{1}{t}}} = t \end{aligned}$$

So, $z \in B(y,\varepsilon)\bigcap A$ and consequently $B(y,\varepsilon)\bigcap A \neq \phi$ for arbitrary $\varepsilon > 0$. Hence, A is dense in S. □

THEOREM 9. *The function $\lambda : S \to [0,\infty]$ is totally discontinuous in S.*

Proof. Let $x = \{x_{mn}\}_{m,n\geqslant 1}$ be any point in S. We take a double sequence $y = \{(mn)^{\alpha}\}_{m,n\geqslant 1}$ with $\alpha > 0$ and $\lambda(x) \neq \frac{1}{\alpha}$. Then,

$$\lambda(y) = \limsup_{m,n\to\infty} \frac{\log(mn)}{\log(mn)^{\alpha}} = \frac{1}{\alpha} \;\text{ i.e., }\; \lambda(x) \neq \lambda(y).$$

We now construct a sequence $\{x^{(i)}\}_{i=1}^{\infty}$ in S as follows:

$$x^{(i)}_{mn} = x_{mn} \quad \text{for} \quad m = 1,2,\ldots,i;\; n = 1,2,\ldots,i.$$

If $(1+i)^{\alpha} \geqslant x_{mn}$, then we take $x^{i}_{mn} = (mn)^{\alpha}$ for all (m,n) when $(m,n) \nleq (i,i)$.

If $(1+i)^{\alpha} < x_{mn}$, then we take $x^{i}_{mn} = x_{mn}$ for $m = i+1, i+2, \ldots, j-1$; $n = i+1, i+2, \ldots, j-1$, where j is the least positive integer so that $(j)^{\alpha} \geqslant x_{mn}$ and we set $x^{i}_{mn} = (mn)^{\alpha}$ for all (m,n) when $(m,n) \nleq (j-1, j-1)$.

It is clear that, the element of the sequence $\{x^{(i)}\}_{i=1}^{\infty}$ belongs to S and $\lim_{i\to\infty} x^{(i)} = x$. Then, $\lambda(x^{(i)}) = \frac{1}{\alpha}$ for $i = 1,2,3,\ldots$ and hence $\lim_{i\to\infty} \lambda(x^{(i)}) = \frac{1}{\alpha} \neq \lambda(x)$. □

REFERENCES

[1] D. K. GANGULY, A. DAFADAR, B. BISWAS, *On Some Properties of a Function Connecting with an infinite Series*, Acta. Math. Univ. Comenianae, **LXXIX**, 2 (2010), 217–223.

[2] D. K. GANGULY, B. BISWAS, *A function on exponential convergence in a Fréchet metric space*, Extracta Mathematicae, **28**, 1 (2013), 49–56.

[3] T. J. BROMWICH, *An Introduction to the Theory of Infinite Series*, Macmillan and Co. Ltd., New York, (1965).

[4] G. PÓLYA UND G. SZEGŐ, *Aufgaben und Lehrsatze Aus Der Analysis*, Springer-Verlag, Berlin, Göttingen, Heidelberg, New York, (1964).

[5] P. KOSTYRKO AND T. ŠALÁT, *On the exponent of convergence*, Rend. Circ. Mat. Palermo Ser. II Tomo, **XXXI**, (1982), 187–194.

[6] A. PRINGSHEIM, *Zur theorie der zweifach unendlichen zahlenfolgen*, Mathematische Annalen, **53**, 3 (1900), 289–321, MR 1511092.

[7] G. H. HARDY, *On the convergence of certain multiple series*, Proc. London Math. Soc., **19** s2-1, **1** (1904), 124–128, MR 1576764.

[8] G. M. ROBISON, *Divergent Double Sequences and Series*, Amer. Math. Soc. Trans. **28**, **1** (1926), 50–73, MR 1501332.

[9] H. J. HAMILTON, *Transformations of Multiple Sequences*, Duke Math. Jour. **2** (1936), 29–60.

[10] G. G. LORENTZ, *A contribution to theory of divergent sequences*, Acta. Math., **80** (1948), 167–190.

[11] D. K. GANGULY, C. DUTTA, *Some properties of a function connected tó a double series*, Bull. Malayasian Math. Sc. Soc. **24** (2001), 177–181.

[12] K. KURATOWSKI, *Topology*, Volume I, Academic Press, (1958).

(Received July 5, 2016)

D. K. Ganguly
Department of Pure Mathematics
University of Calcutta
Kolkata, India
e-mail: gangulydk@yahoo.co.in

B. Biswas
Department of Mathematics
P. N. Das College
Palta, 24 parganas(N), West Bengal, India
e-mail: bablubiswas100@yahoo.com

Alauddin Dafadar
Department of Mathematics
Bhatter College
Dantan, Paschim Medinipur, West Bengal, India
e-mail: alauddindafadar@yahoo.com

15/5/23

Bull. Cal. Math. Soc., **110**, (1) 41–50 (2018)

ON OI-CAUCHY SEQUENCE IN A METRIC ADDITIVE SYSTEM

BABLU BISWAS

(*Received 8 December 2017*)

Abstract. In this paper we study the concept of Cauchy sequences in a linearly ordered additive system associated with the order convergence endowed with a particular metric and we introduce the idea of OI-Cauchy sequences.

1. Introduction. The idea of I-convergence in real numbers was introduced by Kostyrko, (Kostyrko, Salat and Wilczynski, 2000) in 2000 and it is an interesting generalization of statistical convergence. The notion of statistical convergence was introduced in 1951 by Fast (1959) and Schoenberg (1959) independently and it was discussed and developed by several authors viz., (Fridy, 1985, Miller, 1995, Salat, 1980 and Schoenberg, 1959) in different directions. Many authors developed the concept of I-convergences based on the notion of ideal I of subsets of set $\mathbb{N}$ of natural numbers in different spaces (see, (Kostyrko, Macaj, Salat and Sleziak, 2005, Salat, Tripathy and Ziman, 2005 and Tripathy and Tripathy, 2005) etc.).

Recently, the concepts of statistical convergence and I-convergence have been studied in a linearly ordered additive system associated with the order convergence (Aliprantis and Border, 2006 and Birkhoff, 1940) with respect to a particular metric introduced in (Wilcox and Smiley, 1939) by the authors (Biswas and Ganguly, 2016 and Ganguly and Biswas, 2014).

I-Cauchy condition was first introduced and studied by Dems (Dems, (2004–2005)) and also by Gurdal (2004). The concept of I^*-Cauchy sequences have been very recently introduced by Nabiev et al. (2007).

The main purpose of this paper is to examine whether the concepts of I-Cauchy sequence and I^*-Cauchy sequence is extendible in a linearly ordered metric additive system. In this connection we introduce the concepts of OI-Cauchy sequence and OI^*-Cauchy sequence corresponding to the notions of OI-convergence introduced by Biswas and Ganguly, 2016.

2. Definitions and notations. First we recall the definition of natural density of a subset of natural numbers $\mathbb{N}$ and the idea of some generalised convergences.

DEFINITION 2.1 (Niven and Zuckerman, 1980) *If K is a subset of $\mathbb{N}$ then the natural density of K denoted by $\delta(K)$ is defined by,*

$$\delta(K) = \lim_{n\to\infty} \frac{|K_n|}{n},$$

where $K_n = \{k \leq n : k \in K\}$ and $|K_n|$ is the number of elements of K_n.

DEFINITION 2.2 (Fast, 1959) *A sequence $\{x_n\}$ of real numbers is said to be statistically convergent to some number ξ, if for any $\varepsilon > 0$,*

$$\delta(\{k \in \mathbb{N} : |x_k - \xi| \geq \varepsilon\}) = 0.$$

If $\{x_n\}$ is statistically convergent to ξ then we write $st - \lim_n x_n = \xi$.

DEFINITION 2.3 *Let L be a partially ordered set with respect to the order relation $\leq$. L is said to be an additive system if for every two elements $x, y \in L$ there exists a least upper bound $x \vee y$ in L.*

An element θ in L is the null element of L if $x \vee \theta = x$ for all $x \in L$.

If L is a partially ordered set, we say that a sequence $\{x_n\}$ in L is increasing (decreasing) if $x_i \leq x_j$ $(x_i \geq x_j)$ for $i < j$.

NOTE. *To denote a monotone increasing (decreasing) sequence $\{x_n\} \in L$ we use the notation $x_n \uparrow$ $(x_n \downarrow)$. The notation $x_n \downarrow x$ means that $x_n \downarrow$ and $\inf x_n = x$. The meaning of the notation $x_n \uparrow x$ is similar.*

DEFINITION 2.4 (Aliprantis and Border, 2006) *A sequence $\{x_n\}$ in an additive system L is said to be order convergent to $\xi \in L$ if there exists a sequence $\{y_n\}$ of elements of L with $y_n \downarrow \theta$ such that*

$$|x_n - \xi| < y_n \text{ for each } n \in \mathbb{N},$$

where in L, $|x| = x^+ + x^-$ and $x^+ = x \vee \theta$, $x^- = (-x) \vee \theta$

DEFINITION 2.5 (Wilcox and Smiley, 1939) (i) *Let L be an additive system and D be a real valued function defined on L. Then a function γ is defined on L by*

$$\gamma(a, b) = 2D(a \vee b) - D(a) - D(b).$$

$D(a)$ is said to be monotone increasing (decreasing) when

$$D(a) \leq D(b)(D(a) \geq D(b)) \quad for \quad a < b.$$

(ii) *Let L be an additive system and $\gamma(a,b)$ be real valued function defined for every pair $(a,b) \in L$; then define*

$$\Delta(a,b,c) = \tfrac{1}{2}\{\gamma(a,b) + \gamma(b,c) - \gamma(a,c)\} \quad for \quad a,b,c \in L.$$

The following proposition is immediate.

PROPOSITION 2.6 (Wilcox and Smiley, 1939) *If $D(a)$ is a real valued function defined on an additive system L, then for $a, b \in L$*

(i) $\gamma(a,b) = \gamma(b,a)$, $\gamma(a,a) = 0$

(ii) $\Delta(a, a \vee b, b) = 0$

(iii) *$D(a)$ is monotone increasing if and only if $\gamma(a,b) \geq 0$*

(iv) *$D(a)$ is properly monotone increasing if and only if $\gamma(a,b) > 0$ for $a \neq b$.*

NOTE. *If $D(a)$ is monotone increasing and $\Delta(a,b,c) \geq 0$ for every $a,b,c \in L$, then $\gamma(a,b)$ is a metric on L.*

In this connection we mention the following result from the paper (Wilcox and Smiley, 1939).

RESULT 2.7 (Wilcox and Smiley, 1939) *If $D(a)$ is a real valued function defined on an additive system L, then for $a,b,c,d \in L$, $\gamma(a \vee c, b \vee d) \leq \gamma(a,b) + \gamma(c,d)$.*

Now we recall the concept of order statistical convergence in the metric additive system (L, γ).

DEFINITION 2.8 (Ganguly and Biswas, 2014) *A sequence $\{x_n\}_n$ in a metric additive system (L,γ) is said to be order statistically convergent (i.e ost-convergent) to $x \in L$ if, there exists a sequence $\{y_n\}_n$ in L with $y_n \downarrow \theta$ such that*

$$\delta(\{k \in \mathbb{N} : \gamma(x_k, x) \geq D(y_k)\}) = 0,$$

where D is a real valued monotone increasing function on L with $D(\theta) = 0$ and $\Delta(a,b,c) \geq 0$ for all $a,b,c \in L$.

DEFINITION 2.9 (Kostyrko, Salat and Wilczynski, 2000) *Let $X \neq \emptyset$. A family of sets $I \subseteq 2^X$ is said to be an ideal in X provided I satisfies the following conditions:*

(a) $\phi \in I$,

(b) $A \cup B \in I$ *if* $A, B \in I$,

(c) *If* $A \in I$ *and* $B \subseteq A$ *then* $B \in I$.

DEFINITION 2.10 (Kostyrko, Salat and Wilczynski, 2000) *Let* X *be a non-empty set. A non-empty family* $F \subseteq 2^X$ *is said to be a filter on* X *if the following conditions are satisfied*:

(a) $\phi \notin F$,

(b) $A \cap B \in F$ *if* $A, B \in F$,

(c) *If* $A \in F$ *and* $A \subseteq B \subseteq X$ *then* $B \in F$.

An ideal I is said to be non-trivial if $I \neq \phi$ and $X \notin I$.

A non-trivial ideal I is said to be admissible in X if $\{x\} \in I$ for each $x \in X$.

PROPOSITION 2.11 (Kostyrko, Salat and Wilczynski, 2000) *I is a non-trivial ideal in X if and only if the family of sets* $F(I) = \{M \subseteq X : X - M \in I\}$ *is a filter in* X.

It is called the filter associated with the ideal I.

DEFINITION 2.12 (Kostyrko, Salat and Wilczynski, 2000) *Let* I *be a non-trivial ideal of subsets of* $\mathbb{N}$ *and* (X, ρ) *be a metric space. A sequence* $x = \{x_n\}$ *of elements of* X *is said to be* I*-convergent to* $\xi \in X$ *if for each* $\varepsilon > 0$ *the set* $A(\varepsilon) = \{n \in \mathbb{N} : \rho(x_n, \xi) \geq \varepsilon\} \in I$.

If $x = \{x_n\}$ is I-convergent to ξ, then ξ is called the I-limit of the sequence x and we denote it by $I - \lim_{n\to\infty} x_n = \xi$.

DEFINITION 2.13 (Kostyrko, Salat and Wilczynski, 2000) *Let* I *be a non-trivial ideal of subsets of* $\mathbb{N}$ *and* (X, ρ) *be a metric space. A sequence* $x = \{x_n\}$ *of elements of* X *is said to be* I^**-convergent to* $\xi \in X$ *if there exists a set* $M \in F(I)$ *with* $M = \{m_1 < m_2 < m_3 < \cdots\} \subseteq \mathbb{N}$ *such that* $\lim\limits_{n\to\infty} \rho(x_{m_n}, \xi) = 0$.

DEFINITION 2.14 (Kostyrko, Salat and Wilczynski, 2000) *An admissible ideal* I *of subsets of* $\mathbb{N}$ *is said to have* AP*-property if for any sequence* $\{A_1, A_2, A_3, \cdots\}$ *of mutually disjoint sets of* I, *there exists a sequence* $\{B_1, B_2, B_3 ...\}$ *such that for each* $i \in \mathbb{N}$ *the symmetric difference* $A_i \Delta B_i$ *is finite and* $\bigcup\limits_{i=1}^{\infty} B_i \in I$

Here we state the following propositions for subsequent use in our paper.

PROPOSITION 2.15 (Nabiev, Pehlivan and Gurdal, 2007) *Let* I *be an admissible ideal with* AP*-property and* $\{P_n\}$ *be a countable collection of subsets of* $\mathbb{N}$ *such that* $P_i \in F(I)$

for each natural number i. Then there exists a subset P of Natural numbers such that $P \in F(I)$ and $P - P_i$ is finite for all $i \in \mathbb{N}$.

PROPOSITION 2.16 (Biswas and Ganguly, 2016) *If $x = \{x_n\} \in L$ is such that $\lim_{n\to\infty} x_n = \xi$ with respect to the metric γ, then there exists a sequence $\{\alpha_n\} \in L$ with $\alpha_n \downarrow \theta$ such that $\gamma(x_n, \xi) < D(\alpha_n)$, for all $n \in \mathbb{N}$.*

DEFINITION 2.17 (Biswas and Ganguly, 2016) *Let I be a non-trivial ideal of subsets of $\mathbb{N}$ and (L, γ) be a metric additive system. A sequence $x = \{x_n\}$ of elements of L is said to be order ideal convergent (OI-convergent) to $\xi \in L$ if there exists a sequence $y = \{y_n\} \in L$ with $y_n \downarrow \theta$ such that the set $A = \{n \in \mathbb{N} : \gamma(x_n, \xi) \geq D(y_n)\} \in I$, where D is a real valued monotone increasing function defined on L with $D(\theta) = 0$ and $\Delta(a, b, c) \geq 0$ for all $a, b, c \in L$.*

The number ξ is called the order ideal limit (OI-limit) of the sequence $x = \{x_n\}$ and we write $OI - \lim x_n = \xi$.

DEFINITION 2.18 (Biswas and Ganguly, 2016) *Let I be a non-trivial ideal of subsets of $\mathbb{N}$ and (L, γ) be a metric additive system. A sequence $x = \{x_n\}$ of elements of L is said to be OI^*-convergent to $\xi \in L$ if there exists a set $M \in F(I)$ with $M = \{m_1 < m_2 < m_3 < \cdots\}$ and $\lim_{k\to\infty} x_{m_k} = \xi$ with respect to the metric γ.*

Throughout the paper we consider D to be a monotone increasing real valued function with $D(\theta) = 0$ and $\Delta(a, b, c) \geq 0$ for all $a, b, c \in L$.

3. Order ideal Cauchy sequence. Following the idea of OI-convergence we introduce the concept of order ideal Cauchy sequence (OI-Cauchy sequence) in the metric additive system (L, γ) and study some properties related to OI-Cauchy sequences.

DEFINITION 3.1 *For a non-trivial ideal I of subsets of $\mathbb{N}$, a sequence $\{x_n\}$ in (L, γ) is said to be OI-Cauchy if there exists a sequence $\{y_n\}$ in L with $y_n \downarrow \theta$ such that $\{n \in \mathbb{N} : \gamma(x_{n+p}, x_n) \geq D(y_n)\} \in I$, for $p = 1, 2, 3, \cdots$.*

THEOREM 3.2 *Let I be an admissible ideal of subsets of $\mathbb{N}$. Then every OI-convergent sequence in (L, γ) is OI-Cauchy.*

Proof: Let I be an admissible ideal and $\{x_n\}$ be a sequence in L such that $OI - \lim x_n = \xi$, for some $\xi \in L$. Then there is a sequence $\{y_n\}$ in L with $y_n \downarrow \theta$ such that $A = \{n \in \mathbb{N} : \gamma(x_n, \xi) \geq D(y_n)\} \in I$.

We can choose a sequence $\{z_n\}$ in L with $z_n \downarrow \theta$ such that $D(z_n) \geq 2D(y_n)$ for all $n \in \mathbb{N}$. We set, $B = \{n \in \mathbb{N} : \gamma(x_{n+p}, x_n) \geq D(z_n)\}$, for $p = 1, 2, 3, \cdots$.

Since I is an admissible ideal then A^c, the complement of the set A, is non-empty. Let $A^c = \{n_1 < n_2 < n_3 < \cdots\}$.

Then $\gamma(x_{n_k}, \xi) < D(y_{n_k})$ for all $k = 1, 2, 3, \cdots$.

For $p = 1, 2, 3, \cdots$

$$\begin{aligned}\gamma(x_{n_{k+p}}, x_{n_k}) &\leq \gamma(x_{n_{k+p}}, \xi) + \gamma(x_{n_k}, \xi) \\ &< D(y_{n_{k+p}}) + D(y_{n_k}) \\ &\leq 2D(y_{n_k}) \\ &\leq D(z_{n_k}).\end{aligned}$$

So, $n_k \in B^c$ and this implies that $A^c \subseteq B^c$. i.e., $B \subseteq A$ and consequently $B \in I$. Hence, $\{x_n\}$ is an OI-Cauchy sequence.

In the following example we justify an unusual property of OI-Cauchy sequence. In fact a sub sequence of an OI-Cauchy sequence may not be OI-Cauchy.

EXAMPLE 3.3 *Suppose $x, y \in L$ with $x \neq y$ and I is an admissible ideal so that there exists a partition of $\mathbb{N}$ as $\mathbb{N} = A \cup B \cup C$ and A, B, C are pair wise disjoint subsets of $\mathbb{N}$ so that $A \in I$ but $B \notin I, C \notin I$. Let $A = \{m_1 < m_2 < \cdots\}$ and $B \cup C = \{k_1 < k_2 < \cdots\}$.*

Define a sequence $\{x_n\}$ such that

$$\begin{aligned}x_{k_n} &= x, \;\; for\ all\ n \in \mathbb{N} \\ x_{m_n} &= x; \;\; if\ \ n \in A \cup B \\ &= y; \;\; if\ \ n \in C\end{aligned}$$

It is easy to verify that $\{x_n\}$ is OI-Cauchy but $\{x_{m_n}\}$ is not.

DEFINITION 3.4 *Let I be a non-trivial ideal of subsets of $\mathbb{N}$ and (L, γ) be a metric additive system. A sequence $\{x_n\} \in L$ is said to be an OI^*-Cauchy sequence provided there is a set $M = \{m_1 < m_2 < m_3 < \cdots\} \subseteq \mathbb{N}$ with $M \in F(I)$ such that $\{x_{m_k}\}$ is a Cauchy sequence in ordinary sense with respect to the metric γ.*

THEOREM 3.5 *For a non-trivial ideal I, an OI^*-convergent sequence is OI^*-Cauchy in a metric additive system (L, γ).*

Proof: Let I be a non-trivial ideal and $\{x_n\}$ be sequences in L such that $OI^* - \lim x_n = \xi$. Then there exists a set $M = \{m_1 < m_2 < m_3 < \cdots\} \in F(I)$ so that $\lim_{k \to \infty} x_{m_k} = \xi$. Using Proposition 2.16, we can choose a sequence $\{\beta_n\}$ in L with $\beta_n \downarrow \theta$ such that $\gamma(x_{m_k}, \xi) < D(\beta_{m_k})$ for all $k \in \mathbb{N}$. It implies that

$$\gamma(x_{m_{k+p}}, x_{m_k}) \le \gamma(x_{m_{k+p}}, \xi) + \gamma(x_{m_k}, \xi)$$
$$< D(\beta_{m_{k+p}}) + D(\beta_{m_k})$$
$$\le 2D(\beta_{m_k}), \ \textit{for all} \ \ k \in \mathbb{N} \text{ and } p = 1, 2, 3, \cdots.$$

We can choose a sequence $\{z_n\}$ in L with $z_n \downarrow \theta$ such that $D(z_n) \ge 2D(\beta_n)$ for all $n \in \mathbb{N}$ and thus $\gamma(x_{m_{k+p}}, x_{m_k}) < D(z_{m_k})$, for all $k \in \mathbb{N}$ and $p = 1, 2, 3, \cdots$. This shows that $\{x_{m_k}\}$ is a Cauchy sequence and consequently $\{x_n\}$ is an OI^*-Cauchy sequence.

We here examine the relations between OI-Cauchy sequence and OI^*-Cauchy sequence. For this purpose we first proof the following lemma.

LEMMA 3.6 *If* $\{x_n\}$ *is a Cauchy sequence in ordinary sense in* (L, γ) *then there exists a sequence* $\{y_n\} \in L$ *with* $y_n \downarrow \theta$ *such that* $\gamma(x_{n+p}, x_n) < D(y_n)$, *for all* $n \in \mathbb{N}$ *and* $p = 1, 2, \cdots$.

Proof: Let $\{x_n\}$ be a Cauchy sequence in L with respect to the matrix γ. Then for any $\varepsilon > 0$ there exists $m \in \mathbb{N}$ such that $\gamma(x_{n+p}, x_n) < \varepsilon$ for all $n \ge m$ and $p = 1, 2, 3, \cdots$.

Let $\{y_n\}$ be a sequence in L such that $y_n \downarrow \theta$. Then for each y_i there exists a smallest positive integer m_i such that $\gamma(x_{n+p}, x_n) < D(y_i)$ for all $n \ge m_i$, $i = 1, 2, 3, \cdots$. Let $t_i = \sup\{\gamma(x_{i+p}, x_i), p = 1, 2, 3, \cdots\}$.

Choose $z_1 \in L$ such that $D(z_1) \ge \max\{D(y_1), t_1, t_2, \cdots, t_{m_1 - 1}\}$.

Choose $z_2 \in L$ such that,

$$\inf\{\gamma(x_{m_1+p}, x_{m_1}), p \in \mathbb{N}\} \ge D(z_2)$$
$$> \max\{D(y_2), t_{m_1+1}, t_{m_1+2}, , t_{m_2-1}\}.$$

Choose $z_3 \in L$ such that,

$$\inf\{\gamma(x_{m_2+p}, x_{m_2}), p \in \mathbb{N}\} \ge D(z_3)$$
$$> \max\{D(y_3), t_{m_2+1}, t_{m_2+2}, \cdots, t_{m_3-1}\}.$$

and so on.

Now set,
$$\begin{aligned} \alpha_i &= z_1; \ \ i = 1, 2, \cdots, m_1 - 1 \\ &= y_1; \ \ i = m_1 \\ &= z_2; \ \ i = m_1 + 1, m_1 + 2, \cdots, m_2 - 1 \\ &= y_2; \ \ i = m_2 \\ &\cdots\cdots\cdots\cdots \end{aligned}$$

Then, $\gamma(x_{n+p}, x_n) < D(\alpha_n)$, for all $n \in \mathbb{N}$ and $p = 1, 2, 3, \cdots$ with $\alpha_n \downarrow \theta$.

THEOREM 3.7 *Let I be an admissible ideal of subsets of $\mathbb{N}$. If a sequence $x = \{x_n\}$ in L is OI^*-Cauchy then it is OI-Cauchy.*

Proof: Let I be an admissible ideal and $\{x_n\}$ be an OI^*-Cauchy sequence in L. Then there is a set $M = \{m_1 < m_2 < m_3 < \cdots\} \subseteq \mathbb{N}$ with $M \in F(I)$ such that $\{x_{m_n}\}$ is a Cauchy sequence in ordinary sense with respect to the metric γ. By the Lemma 3.6, there exists a sequence $\{y_n\} \in L$ with $y_n \downarrow \theta$ such that $\gamma(x_{m_{n+p}}, x_{m_n}) < D(y_n)$ for all $n \in \mathbb{N}$ and $p = 1, 2, \cdots$.

Consider a sequence $\{z_n\}$ in L as follows.

$$\begin{aligned} z_i &= y_1; \ \ i = 1, 2, \cdots, m_1 \\ &= y_2; \ \ i = m_1 + 1, m_1 + 2, \cdots, m_2 \\ &\cdots\cdots\cdots\cdots \end{aligned}$$

Clearly, $\gamma(x_{m_{n+p}}, x_{m_n}) < D(z_{m_n})$ for all $n \in \mathbb{N}$ and $p = 1, 2, \cdots$.

So, $\{n \in \mathbb{N} : \gamma(x_{n+p}, x_n) \geq D(z_n)\} \subseteq \mathbb{N} - M \in I$, for $p = 1, 2, 3 \cdots$ and thus $\{x_n\}$ is an OI-Cauchy sequence.

NOTE. *In the following example we see that that the converse of the theorem is not true.*

EXAMPLE 3.8 *Let $\mathbb{P}$ be the set of all primes. For $p \in \mathbb{P}$ consider the set $N_p = \{p, p^2, p^3, \cdots\}$ and $N_1 = \mathbb{N} - \bigcup_{p \in \mathbb{P}} N_p$.*

Let $I = \{A \subseteq \mathbb{N} : A$ intersects only a finite number of N_p's. If L has an accumulation point $\xi \in L$, then there exists a sequence $\{x_n\}$ in L so that $x_n \to \xi$ with respect to γ. Using Proposition 2.16, we can choose a sequence $\{\alpha_n\}$ in L with $\alpha_n \downarrow \theta$ so that

$$\gamma(x_n, \xi) < D(\alpha_n), \ \textit{for all } n \in \mathbb{N}.$$

Define a sequence $\{y_n\}$ in L with $y_n = x_j$ if $n \in N_j$ where j is either 1 or a prime number.

In Example (3.16, (Biswas and Ganguly, 2016)), *it is proved that $\{y_n\}$ is OI-convergent and hence it is OI-Cauchy.*

If possible let $\{y_n\}$ be an OI^*-Cauchy sequence. Then there is a set $M = \{m_1 < m_2 < m_3 < \cdots\} \subseteq \mathbb{N}$ with $M \in F(I)$ such that

$$\gamma(y_{m_{k+p}}, y_{m_k}) < D(\beta_{m_k}) \ \textit{for all } k \in \mathbb{N} \textit{ and } p = 1, 2, \cdots$$

for some sequence $\{\beta_n\}$ with $\beta_n \downarrow \theta$.

Let $H = \mathbb{N} - M$, then $H \in I$ and there exist prime numbers $p_1, p_2, \cdots, p_r$ such that $H \subseteq N_1 \cup N_1 \cup \cdots \cup N_{p_r}$. There exists natural number $s > p_r$ so that $N_s \subseteq \mathbb{N} - H = M$ and $m_k \in N_s$ implies that $y_{m_k} = x_s$ for infinitely many k. If $m_k \notin N_s$ for infinitely many k then there exists $t > p_r$ with $N_t \subset \mathbb{N}$ such that $m_k \in N_t$. This implies that $y_{m_k} = x_t$ for infinitely many k. Thus,

$$\gamma(y_{m_{k+p}}, y_{m_k}) = \gamma(x_s, x_t) > 0$$

for infinitely many k which is a contradiction and consequently $\{y_n\}$ is not OI^*-Cauchy.

In the following we provide the condition for coincidence of two types of Cauchy Sequences.

THEOREM 3.9 *If I is an admissible ideal with (AP)-property then a sequence $\{x_n\}$ in L is OI-Cauchy if and only if it is an OI^*-Cauchy sequence.*

Proof: Let $\{x_n\}$ be an OI-Cauchy sequence in L. Then there is a sequence $\{y_n\}$ in L with $y_n \downarrow \theta$ such that

$$\{n \in \mathbb{N} : \gamma(x_{n+p}, x_n) \geq D(y_n)\} \in I \text{ for } p = 1, 2, 3 \cdots$$

Let $B_i = \{p \in \mathbb{N} : \gamma(x_{i+p}, x_i) < D(y_i)\}$. Clearly, $B_i \in F(I)$ for all $i \in \mathbb{N}$. By the Proposition 2.15, there is a set $B \in F(I)$ so that $B - B_i$ is finite for all $i \in \mathbb{N}$.

Let $\varepsilon > 0$. We can choose a natural number j for which $D(y_j) < \varepsilon/2$. Since $B - B_j$ is finite then for $n \in B$, there exists a natural number k_j such that $n \in B_j$ for $n > k_j$. So, $\gamma(x_{j+n}, x_j) < D(y_j) < \varepsilon/2$ for $n > k_j$. This implies that $\gamma(x_{j+n}, x_{j+m}) < \gamma(x_{j+n}, x_j) + \gamma(x_{j+m}, x_j) < \varepsilon/2 + \varepsilon/2 = \varepsilon$ for $n, m > k_j$. i.e., $\gamma(x_n, x_m) < \epsilon$ for $n, m > j + k_j$ and $n, m \in B$. Thus, $\{x_n\}_{n \in B}$ is a Cauchy sequence in ordinary sense with respect γ and consequently $\{x_n\}$ is an OI^*-Cauchy sequence.

The converse part follows from the Theorem 3.7. Hence, the theorem is proved.

Acknowledgement. The author is grateful to the reviewer for his careful reading and making some useful corrections which improved the presentation of the paper. The researcher also gratefully acknowledge the financial support from UGC, New Delhi, India.

References

Aliprantis, C.D. and **Border, Kim C.** (2006) : Infinite Dimensional Analysis, Springer Berlin Heidelberg, (Third Edition), New York.

Biswas, B. and **Ganguly, D.K.** (2016) : On Order Ideal Convergence in a Metric Additive System, *J. Ana. Num. Theor.*, **4**, No. 2, 159.

Birkhoff, G. (1940) : Lattice Theory, *Amer. Math. Soc. Colloq. Pubi.*, vol. **XXV** (First Edition), New York.

Dems, K. (2004–2005) : On I-Cauchy sequences, Real Anal. Exchange, **30**(1), 123.

Fast, H. (1959) : Sur la convergence statistique, *Colloq. Math.*, **2**, 241.

Fridy, J.A. (1985) : On statistical convergence, *Analysis*, **5** (no. 4), 301.

Ganguly, D.K. and **Biswas, Bablu** (2014) : Order statistical convergence in a metric space, *Investigations in Mathematical Sciences*, **4**(2), 11.

Gurdal, M. (2004) : Some type of convergence, Doctoral Diss., S. Demirel Univ., Isparta.

Kostyrko, P., Macaj, M., Salat, T. and **Sleziak, M.** (2005) : I-convergence and External I-limit Points, *Math. Slovaca*, **55**, 443.

Kostyrko, P., Salat, T. and **Wilczynski, W.** (2000) : I-convergence, *Real Anal. Exchange*, **26(2)**, 669.

Miller, H.I. (1995) : A measure theoretical subsequence characterization of statistical convergence, *Trans. Amer. Math. Soc.*, **374** (no. 5), 1811.

Nabiev, A., Pehlivan, S. and **Gurdal, M.** (2007) : On I-Cauchy sequences, *Taiwanese J. Math.*, **11** (2), 569.

Niven, I. and **Zuckerman, H.S.** (1980) : An Introduction to the Theory of Numbers, John Wiley & Sons, New York, USA, 4-th edition, Chapter-11.

Salat, T. (1980) : On statistically convergent sequences of real numbers, *Math. Slovaca*, **30**, 139.

Salat, T., Tripathy, B.C. and **Ziman, M.** (2005) : On I-convergence field, *Ital. J. Pure Appl. Math.*, **17**, 45.

Schoenberg, I.J. (1959) : The integrability of certain functions and related summability methods, *Amer. Math. Monthly*, **66** (no. 5), 361.

Steinhaus, H. (1951) : Sur la convergence ordinaire et la convergence asymptotique, *Colloq. Math.*, **2**, 73.

Tripathy, B.K. and **Tripathy, B.C.** (2005) : On I-convergent double sequence., *Soochow J. of Math.*, **31** (no.4), 549.

Wilcox, L.R. and **Smiley, M.F.** (1939) : Metric Lattices, *Ann. of Math.*, **40** (no. 2), 309.

P.N. DAS COLLEGE
PATLA

E-MAIL : bablubiswas100@yahoo.com

15/5/23

Natural Sciences
Publishing

Login New user?

Journal of Analysis & Number Theory

An International Journal

Home | About us | News | Journals | Conferences | Contact us | Books

JANT Home
Aim and Scope
For Reviewers
For Authors
Editorial Board
Publication Ethics
Submit an Article
Content
Forthcoming Papers
Subscription

Content

Volumes > Vol. 4 > No. 2

On Order Ideal Convergence in a Metric Additive System

PP: 159-164

doi:10.18576/jant/040212

Author(s)
Bablu Biswas, D. K. Ganguly,

Abstract
In this paper we study the concept of Ideal convergence in a linearly ordered additive system associated with the order convergence endowed with a particular metric and we introduce the idea of order ideal convergence.

Home About us News Journals Conferences Contact us

15/5/23

J. Ana. Num. Theor. 4, No. 2, 159-164 (2016)

Journal of Analysis & Number Theory
An International Journal

http://dx.doi.org/10.18576/jant/040212

On Order Ideal Convergence in a Metric Additive System

Bablu Biswas[1,*] *and D. K. Ganguly*[2]

[1] Department of Mathematics, P. N. Das College, Palta, North 24 Parganas, West Bengal, India.
[2] Department of Pure Mathematics, University of Calcutta, 35, Circular road, Kolkata-700019, India.

Received: 2 Feb. 2016, Revised: 2 Apr. 2016, Accepted: 9 Apr. 2016
Published online: 1 Jul. 2016

Abstract: In this paper we study the concept of Ideal convergence in a linearly ordered additive system associated with the order convergence endowed with a particular metric and we introduce the idea of order ideal convergence.

Keywords: Additive system, order convergence, natural density, statistical convergence, ideal convergence.

1 Introduction

The idea of I-convergence in real numbers was introduced by Kostyrko, Šalát and Wilczyński [9] in 2000 and it is an interesting generalization of statistical convergence. The notion of statistical convergence was introduced in 1951 by Fast [5] and Schoenberg [14] independently and it was discussed and developed by several authors viz. [6,10, 15]. Many authors [4,8,13,16,17,18,19,20] developed the concept of I-convergence based on the notion of ideal I of subsets of the set $\mathbb{N}$ of natural numbers in different spaces.

Recently the concept of statistical convergence has been studied in a linearly ordered additive system associated with the order convergence with respect to a particular metric in [3].

The order convergence is one of the main concept used in this paper and it was described and developed by many authors including [1,2,7,12].

The main purpose of this paper is to examine whether the concept of I-convergence is extendable in a linearly ordered metric additive system mentioned in [3] and we introduce the concept of OI-convergence and study some basic properties of this convergence.

2 Definitions and notations

First we recall the definition of natural density of a subset of natural numbers $\mathbb{N}$ and the idea of statistical convergence.

Definition 2.1. ([11]) If K is a subset of the set of positive integers $\mathbb{N}$ then the natural density of K is defined by,

$$\delta(K) = lim_{n\to\infty}\frac{|K_n|}{n}, \text{ where}$$

$K_n = \{k \leq n : k \in K\}$ and $|K_n|$ is the number of elements of K_n.

Definition 2.2. ([5]) A sequence $\{x_n\}$ of real numbers is said to be statistically convergent to some number ξ, if for any $\varepsilon > 0$,

$$\delta(\{k \in \mathbb{N} : |x_k - \xi| \geq \varepsilon\}) = 0.$$

If $\{x_n\}$ is statistically convergent to ξ, then we write $st - lim_n x_n = \xi$.

We now mention the idea of order convergence and a particular metric γ in a linearly ordered additive system L introduced in the paper [21] and also recall definition of an ideal.

Definition 2.3. Let L be a set of the elements $x, y, z, \ldots$ and $\leq$ is a binary relation defined for all pairs (x,y) for $x,y \in L$.

We say that L is partially ordered set with respect to $\leq$, if for all $x,y,z \in L$

(i) $x \leq x$ for all $x \in L$,

(ii) $x \leq y$ and $y \leq x$ implies $x = y$ and

(iii) $x \leq y$ and $y \leq z$ implies that $x \leq z$.

If $x \leq y$ and $x \neq y$, we write $x < y$. The relation $x \leq y$ is also written as $y \geq x$. Similarly, $x < y$ is also written as $y > x$.

A partially ordered set L is said to be a lattice if every two

* Corresponding author e-mail: bablubiswas100@yahoo.com

elements $x, y \in L$ possess a least upper bound $x \vee y \in L$ and a greatest lower bound $x \wedge y \in L$.
L is said to be an additive system if for every two elements $x, y \in L$ there exists a least upper bound $x \vee y$ in L and L is said to be a multiplicative system if for every two elements $x, y \in L$ there exists a greatest lower bound $x \wedge y$ in L.

An element θ in L is the null element of L if $x \vee \theta = x$ for all $x \in L$.
If L is a partially ordered set, we say that a sequence $\{x_i\}$ is increasing (decreasing) if $x_i \leq x_j$ $(a_i \geq a_j)$ for $i < j$.

Note 2.4. To denote a monotone increasing (decreasing) sequence $\{x_n\} \in L$ we use the notation $x_n \uparrow$ $(x_n \downarrow)$. The notation $x_n \downarrow x$ means that $x_n \downarrow$ and $\inf x_n = x$. The meaning of the notation $x_n \uparrow x$ is similar.

Definition 2.5. ([7]) A sequence $\{x_n\}$ in an additive system L is said to be order convergent (O-convergent) to $\xi \in L$ if there exists a sequence $\{y_n\}$ of elements of L with $y_n \downarrow \theta$ such that

$$|x_n - \xi| < y_n \text{ for each } n \in \mathbb{N},$$

where in L, $|x| = x^+ + x^-$ and $x^+ = x \vee \theta, x^- = (-x) \vee \theta$.

Definition 2.6. [21] (i) Let L be an additive system and D be a real valued function defined on L. Then a function γ is defined on L by

$$\gamma(a,b) = 2D(a \vee b) - D(a) - D(b).$$

$D(a)$ is said to be monotone increasing (decreasing) when

$$D(a) \leq D(b)(D(a) \geq D(b)) \text{ for } a < b.$$

(ii) Let L be an additive system and $\gamma(a,b)$ be real valued function defined for every pair $(a,b) \in L$; then define

$$\Delta(a,b,c) = \tfrac{1}{2}\{\gamma(a,b) + \gamma(b,c) - \gamma(a,c)\} \text{ for } a,b,c \in L.$$

The following proposition is immediate.

Proposition 2.7. ([21]) If $D(a)$ is a real valued function defined on an additive system L, then for $a, b \in L$
(i) $D(a) - D(b) = \gamma(a,b)$ if $a \geq b$
(ii) If $D(a)$ is monotone increasing, then $|D(a) - D(b)| \leq \gamma(a,b)$
(iii) $\gamma(a,b) = \gamma(b,a)$, $\gamma(a,a) = 0$
(iv) $\Delta(a, a \vee b, b) = 0$
(v) $D(a)$ is monotone increasing if and only if $\gamma(a,b) \geq 0$
(vi) $D(a)$ is properly monotone increasing if and only if $\gamma(a,b) > 0$ for $a \neq b$.

Note 2.8. If $D(a)$ is monotone increasing and $\Delta(a,b,c) \geq 0$ for every $a,b,c \in L$, then $\gamma(a,b)$ is a metric on L.
In this connection we mention the following result from the paper [21].

Result 2.9. If $D(a)$ is a real valued function defined on an additive system L, then
(A) $\Delta(a,b,c) \geq 0$ for every $a,b,c \in L$ implies the following equivalent statements .
(i) $\gamma(a \vee c, b \vee c) \leq \gamma(a,b)$ for all $a, b \in L$
(ii) $\gamma(a \vee c, b \vee c) \leq \gamma(a,b)$ for all $b \leq a$
(iii) $D(a \vee c) + D(b) \leq D(a) + D(c \vee b)$ for $b \leq a$
(iv) $\gamma(a \vee c, b \vee d) \leq \gamma(a,b) + \gamma(c,d)$
(B) If $D(a)$ is monotone increasing, then $\Delta(a,b,c) \geq 0$ if and only if one of the equivalent statements $(i) - (iv)$ holds.

Here we mention the concept of order statistical convergence in the metric additive system (L, γ).

Definition 2.10. [3] A sequence $\{x_n\}_n$ in a metric additive system (L, γ) is said to be order statistically convergent (i.e ost-convergent) to $x \in L$ if, there exists a sequence $\{y_n\}_n$ in L with $y_n \downarrow \theta$ such that

$$\delta(\{k \in \mathbb{N} : \gamma(x_k, x) \geq D(y_k)\}) = 0,$$

where D is a real valued monotone increasing function on L with $D(\theta) = 0$ and $\Delta(a,b,c) \geq 0$ for all $a,b,c \in L$.

We now recall the concept of an ideal and filter of a non-empty set and I-convergence of a sequence.

Definition 2.11. [9] Let $X \neq \emptyset$. A family of sets $I \subseteq 2^X$ is said to be an ideal in X provided I satisfies the following conditions:
(a) $\phi \in I$,
(b) $A \cup B \in I$ if $A, B \in I$,
(c) If $A \in I$ and $B \subseteq A$ then $B \in I$.

Definition: 2.12. [9] Let X be a non-empty set. A non-empty family $F \subseteq 2^X$ is said to be a filter on X if the following conditions are satisfied:
(a) $\phi \notin F$,
(b) $A \cap B \in F$ if $A, B \in F$,
(c) If $A \in F$ and $A \subseteq B \subseteq X$ then $B \in F$.

An ideal I is said to be non-trivial if $I \neq \phi$ and $X \notin I$.
A non-trivial ideal I is said to be admissible in X if $\{x\} \in I$ for each $x \in X$.

Lemma 2.13. [9] I is a non-trivial ideal in X if and only if the family of sets $F(I) = \{M \subseteq X : X - M \in I\}$ is a filter in X.
It is called the filter associated with the ideal I.

Definition 2.14. [9] Let I be a non-trivial ideal of subsets of $\mathbb{N}$, the set of natural numbers and (X, ρ) be a metric space. A sequence $x = \{x_n\}$ of elements of X is said to be I-convergent to $\xi \in X$ if for each $\varepsilon > 0$ the set $A(\varepsilon) = \{n \in \mathbb{N} : \rho(x_n, \xi) \geq \varepsilon\} \in I$.

If $x = \{x_n\}$ is I-convergent to ξ, then ξ is called the I-limit of the sequence x and we denote it by $I - \lim_{n\to\infty} x_n = \xi$.

Definition 2.15. [9] Let I be a non-trivial ideal of subsets of $\mathbb{N}$ and (X,ρ) be a metric space. A sequence $x = \{x_n\}$ of elements of X is said to be I^*-convergent to $\xi \in X$ if there exists a set $M \in F(I)$ with $M = \{m_1 < m_2 < m_3 < ....\} \subseteq \mathbb{N}$ such that $\lim_{n\to\infty} \rho(x_{m_n}, \xi) = 0$.

Definition: 2.16. [9] An admissible ideal I of subsets of $\mathbb{N}$ is said to have AP-property if for any sequence $\{A_1, A_2, A_3, ...\}$ of mutually disjoint sets of I, there exists a sequence $\{B_1, B_2, B_3...\}$ such that for each $i \in \mathbb{N}$ the symmetric difference $A_i \Delta B_i$ is finite and $\cup_{i=1}^{\infty} B_i \in I$.

3 Order ideal convergence

Following the idea of ost-convergence we introduce the concept of order ideal convergence in the metric additive system (L,γ) where γ is a metric defined in [21] and study some general properties related to this convergence.

Definition 3.1. Let I be a non-trivial ideal of subsets of $\mathbb{N}$ and (L,γ) be a metric additive system. A sequence $x = \{x_n\}$ of elements of L is said to be order ideal convergent (OI-convergent) to $\xi \in L$ if there exists a sequence $y = \{y_n\} \in L$ with $y_n \downarrow \theta$ such that the set $A = \{n \in \mathbb{N} : \gamma(x_n,\xi) \geq D(y_n)\} \in I$, where D is a real valued monotone increasing function defined on L with $D(\theta) = 0$ and $\Delta(a,b,c) \geq 0$ for all $a,b,c \in L$.

The number ξ is called the order ideal limit (OI-limit) of the sequence $x = \{x_n\}$ and we write $OI - \lim x_n = \xi$.
Throughout the paper we consider D to be a monotone increasing real valued function with $D(\theta) = 0$ and $\Delta(a,b,c) \geq 0$ for all $a,b,c \in L$.

Note 3.2. From the definition of OI-convergence it is clear that an OI-convergent sequence is I-convergent. In particular if D be an identity map and $L = \mathbb{R}$, then γ becomes the usual metric on $\mathbb{R}$. In this case OI-convergence is equivalent to the I-convergence of real numbers.

Example 3.3. If I_f is the family of all finite subsets of $\mathbb{N}$ then I_f is an admissible ideal on $\mathbb{N}$ and the OI-convergence coincides with the ordinary convergence.

Example 3.4. If $I_\delta = \{A \subseteq \mathbb{N} : \delta(A) = 0\}$ then I_δ is an admissible ideal in $\mathbb{N}$ and the OI-convergence coincides with the order statistical convergence.

We give an example of a sequence which is OI-convergent but not convergent in (L,γ) in ordinary sense.

Example 3.5. Consider the ideal I_f and let $L = \mathbb{R}$ with D as the identity mapping. Then clearly (L,γ) becomes the usual metric space.
Consider a sequence $\{x_n\}$ in $\mathbb{R}$ as follows:

$$x_n = \begin{cases} 1, \text{ if } n \text{ is a square} \\ 0, \text{ otherwise} \end{cases}$$

Let $\{y_n\}$ be a sequence in $\mathbb{R}$ such that $y_n = \frac{1}{n}$. Then $\{n \in \mathbb{N} : \gamma(x_n, 0) \geq D(y_n)\} \in I$. So $OI - \lim x_n = 0$, but $\{x_n\}$ is not convergent with respect to the metric γ.

Theorem 3.6. If I is a non-trivial ideal, then OI-limit of any sequence if exists, is unique.

Proof: Let $x = \{x_n\}$ be a sequence in L such that x is OI-convergent to ξ as well as ν and suppose $\xi \neq \nu$. Let $\varepsilon = \frac{1}{3}\gamma(\xi,\nu)$. Then $B(\xi,\varepsilon) \cap B(\nu,\varepsilon) = \phi$ where $B(\xi,\varepsilon)$ is the open ball with centre at ξ and ε as the radius.
Since x is OI-convergent to both ξ and ν, then there exists two sequences $\{y_n^{(1)}\}$ and $\{y_n^{(2)}\}$ in L with $y_n^{(1)} \downarrow \theta$ and $y_n^{(2)} \downarrow \theta$ such that $\{k \in \mathbb{N} : \gamma(x_k,\xi) \geq D(y_k^{(1)})\} \in I$ and $\{k \in \mathbb{N} : \gamma(x_k,\nu) \geq D(y_k^{(2)})\} \in I$. Now $y_n^{(1)} \downarrow 0$ and $y_n^{(2)} \downarrow 0$ implies that there exists $n_0 \in \mathbb{N}$ such that $D(y_n^{(1)}) < \varepsilon/2$ and $D(y_n^{(2)}) < \varepsilon/2$ for all $n \geq n_0$. Then for $k \geq n_0$, $\{k \in \mathbb{N} : \gamma(x_k,\xi) < D(y_k^{(1)})\} \subseteq \{k \in \mathbb{N} : \gamma(x_k,\xi) < \varepsilon/2\}$. So for $k \geq n_0$, $A = \{k \in \mathbb{N} : \gamma(x_k,\xi) < \varepsilon/2\} \in F(I)$ since $\{k \in \mathbb{N} : \gamma(x_k,\xi) < D(y_k^{(1)})\} \in F(I)$. Similarly for $k \geq n_0$, $B = \{k \in \mathbb{N} : \gamma(x_k,\nu) < \varepsilon/2\} \in F(I)$. Thus for $k \geq n_0$, $A \cap B \in F(I)$ and $A \cap B \neq \phi$ which is a contradiction and hence the proof.

Lemma 3.7. If $x = \{x_n\} \in L$ is such that $\lim_{n\to\infty} x_n = \xi$ with respect to the metric γ, then there exists a sequence $\{\alpha_n\} \in L$ with $\alpha_n \downarrow \theta$ such that $\gamma(x_n,\xi) < D(\alpha_n)$, for all $n \in \mathbb{N}$.

Proof: Since $\lim_{n\to\infty} x_n = \xi$, then for $\varepsilon > 0$ there exists $m \in \mathbb{N}$ such that $\gamma(x_n,\xi) < \varepsilon$ for all $n \geq m$.
Let $\{y_n\}$ be a sequence in L such that $y_n \downarrow \theta$. Then for each y_i there exists a smallest positive integer m_i such that $\gamma(x_n,\xi) < D(y_i)$ for all $n \geq m_i$, $i = 1,2,3,...$
Choose $z_1 \in L$ such that, $D(z_1) \geq max\{D(y_1), \gamma(x_1,\xi), \gamma(x_2,\xi), ..., \gamma(x_{m_1-1},\xi)\}$,

Choose $z_2 \in L$ such that, $\gamma(x_{m_1},\xi) \geq D(z_2) > max\{D(y_2), \gamma(x_{m_1+1},\xi), \gamma(x_{m_1+2},\xi), ..., \gamma(x_{m_2-1},\xi)\}$,

Choose $z_3 \in L$ such that, $\gamma(x_{m_2},\xi) \geq D(z_3) > max\{D(y_3), \gamma(x_{m_2+1},\xi), \gamma(x_{m_2+2},\xi), ..., \gamma(x_{m_3-1},\xi)\}$,

and so on.
Now set,

$$\begin{aligned}\alpha_i &= z_1;\ i = 1,2,\ldots\ldots,m_1-1\\ &= y_1;\ i = m_1\\ &= z_2;\ i = m_1+1,m_1+2,\ldots\ldots,m_2-1\\ &= y_2;\ i = m_2\\ &\ldots\ \ldots\ldots\ldots\ldots\end{aligned}$$

Then

$$\gamma(x_n,\xi) < D(\alpha_n), \text{ for all } n\in\mathbb{N} \text{ and } \alpha_n\downarrow\theta.$$

Theorem 3.8. If I is a non-trivial ideal and $x=\{x_n\}\in L$ be such that $\lim_{n\to\infty}x_n=\xi$ with respect to the metric γ, then $OI-\lim x_n=\xi$.

Proof: Let $x=\{x_n\}\in L$ be a sequence such that $\lim_{n\to\infty}x_n=\xi$ with respect to the metric γ. Then by Lemma 3.7 there exists a sequence $\{\alpha_n\}\in L$ with $\alpha_n\downarrow\theta$ such that $\gamma(x_n,\xi)<D(\alpha_n)$, for all $n\in\mathbb{N}$.
Then $\{n\in\mathbb{N}:\gamma(x_n,\xi)\geq D(\alpha_n)\}=\phi\in I$. So, $OI-\lim x_n=\xi$.

Theorem 3.9. If I is a non-trivial ideal and if $\{x_n\}$ and $\{y_n\}$ are two sequences in L such that $OI-\lim x_n=\xi$ and $OI-\lim y_n=\nu$, then $OI-\lim(x_n\vee y_n)=\xi\vee\nu$.

Proof: Since $OI-\lim x_n=\xi$ and $OI-\lim y_n=\nu$, then there exists sequences $\{\alpha_n\}$ and $\{\beta_n\}$ in L with $\alpha_n\downarrow\theta$ and $\beta_n\downarrow\theta$ such that
$A=\{n\in\mathbb{N}:\gamma(x_n,\xi)\geq D(\alpha_n)\}\in I$ and $B=\{n\in\mathbb{N}:\gamma(y_n,\nu)\geq D(\beta_n)\}\in I$.
Let $p\in A^c\cap B^c$. Clearly $\gamma(x_p,\xi)<D(\alpha_p)$ and $\gamma(y_p,\nu)<D(\beta_p)$. Since D is an increasing function, then by using Result 2.9(B) we have

$$\gamma(x_p\vee y_p,\xi\vee\nu)\leq\gamma(x_p,\xi)+\gamma(y_p,\nu)<D(\alpha_p)+D(\beta_p).$$

Since $\alpha_n\downarrow\theta$ and $\beta_n\downarrow\theta$ we can consider a sequence $\{\delta_n\}\in L$ with $\delta_n\downarrow\theta$ and $D(\delta_n)\geq D(\alpha_n)+D(\beta_n)$ for all $n\in\mathbb{N}$. Then $\gamma(x_p\vee y_p,\xi\vee\nu)<D(\delta_p)$.
Let $C=\{n\in\mathbb{N}:\gamma(x_n\vee y_n,\xi\vee\nu)\geq D(\delta_n)\}$. Then $p\in C^c$ and hence $A^c\cap B^c\subseteq C^c$. This implies that $C\subseteq A\cup B\in I$ since $A,B\in I$ and consequently $OI-\lim(x_n\vee y_n)=\xi\vee\nu$.

Definition 3.10 Let I be a non-trivial ideal of subsets of $\mathbb{N}$ and (L,γ) be a metric additive system. A sequence $x=\{x_n\}$ of elements in L is said to be order ideally bounded(i.e. OI-bounded) in L if there exists $B\in\mathbb{R}$ such that the set $\{n\in\mathbb{N}:D(x_n)\geq B\}\in I$.

Theorem 3.11. Let I be a non-trivial ideal of subsets of $\mathbb{N}$. An OI-convergent sequence in the metric additive system (L,γ) is OI-bounded.

Proof: Let $x=\{x_n\}$ be a sequence in L such that $OI-\lim x_n=\xi$. Then there exists a sequence $\{y_n\}$ in L with $y_n\downarrow\theta$ such that $\{n\in\mathbb{N}:\gamma(x_n,\xi)\geq D(y_n)\}\in I$. i.e. $A=\{n\in\mathbb{N}:\gamma(x_n,\xi)<D(y_n)\}\in F(I)$.
Let $p\in A$. Then $\gamma(x_p,\xi)<D(y_p)$
i.e., $2D(x_p\vee\xi)-D(x_p)-D(\xi)<D(y_p)$. Then $D(x_p)\leq 2D(x_p\vee\xi)-D(x_p)<D(y_p)+D(\xi)$.
Since $y_n\downarrow\theta$, then $D(y_n)\downarrow 0$ and consequently, $\{D(y_n)\}$ is bounded and we can choose a real number M such that $M=\sup\{D(y_p):p\in A\}$. Clearly $D(x_p)<D(\xi)+M$ and so $A\subseteq\{k\in\mathbb{N}:D(x_k)<D(\xi)+M\}\in F(I)$. Hence the proof.

Theorem 3.12 Let I be an admissible ideal of subsets of $\mathbb{N}$ and (L,γ) be a metric additive system. If I contains an infinite set, then there exists an OI-convergent sequence $\{x_n\}$ in L, which has subsequence, which does not converge to the same limit.

Proof: Let A be an infinite set in I and $A=\{n_1,n_2,n_3,\ldots\}$ with $n_1<n_2<n_3<\ldots\ldots$
Again let $B=\mathbb{N}-A=\{m_1,m_2,m_3,\ldots.\}$ with $m_1<m_2<m_3<\ldots.$
Since I is admissible then B is also an infinite set.
Let us choose $\eta,\xi\in L$ such that $\eta\neq\xi$ and consider a sequence $\{x_n\}\in L$ such that

$$\begin{aligned}x_k &= \eta;\ if\ k\in A,\\ &= \xi;\ if\ k\in B.\end{aligned}$$

We choose a sequence $\{y_n\}$ of non-null elements in L such that $y_n\downarrow\theta$. This implies that $\{n\in\mathbb{N}:\gamma(x_n,\xi)\geq D(y_n)\}\subseteq A\in I$. Clearly, $OI-\lim x_k=\xi$. But $\{n_k\in\mathbb{N}:\gamma(x_{n_k},\eta)\geq D(y_n)\}=\phi\in I$ and consequently the subsequence $\{x_{n_k}\}$ is OI-convergent to η.

Theorem 3.13. Let I be an admissible ideal of subsets of natural numbers and each sequence $x=\{x_n\}$ in L has a subsequence which is OI-convergent to ξ, then x is OI-convergent to ξ.

Proof: Let $x=\{x_n\}$ be a sequence in L such that each subsequence of x has a subsequence that is OI-convergent to ξ but $OI-\lim x_n\neq\xi$.
Then for each $\{y_n\}\in L$ with $y_n\downarrow\theta$, $A=\{n\in\mathbb{N}:\gamma(x_n,\xi)\geq D(y_n)\}\notin I$. i.e. $A\in F(I)$ and A is an infinite set since I is admissible.
Let $A=\{n_1<n_2<n_3<\ldots\}$ and $\{x_{n_k}\}$ be a subsequence of x. Then if we choose any subsequence $\{x_{p_k}\}$ of $\{x_{n_k}\}$, then clearly
$\{p_k\in\mathbb{N}:\gamma(x_{p_k},\xi)\geq D(y_{p_k})\}\notin I$ which is a contradiction. Therefore, $OI-\lim x_n=\xi$.

Definition 3.14. Let I be a non-trivial ideal of subsets of $\mathbb{N}$ and (L,γ) be a metric additive system. A sequence $x=\{x_n\}$ of elements of L is said to be OI^*-convergent to $\xi\in L$ if there exists a set $M\in F(I)$ with $M=\{m_1<m_2<m_3<\ldots..\}$ and $\lim_{k\to\infty}x_{m_k}=\xi$ with respect to the metric γ.

Theorem 3.15. Let I be a non-trivial ideal of subsets of $\mathbb{N}$. If $\{x_n\}$ is a sequence in L such that $OI^* - \lim x_n = \xi$, then $OI - \lim x_n = \xi$.

Proof: Let $OI^* - \lim x_n = \xi$. Then there exists $M \in F(I)$ with $M = \{m_1 < m_2 < m_3 < .....\}$ such that $\lim_{k\to\infty} x_{m_k} = \xi$. Then we can choose $\{\beta_n\}$ in L with $\beta_n \downarrow \theta$ by using Lemma 3.7 such that $\gamma(x_{m_k}, \xi) < D(\beta_{m_k})$, for all $k \in \mathbb{N}$.
Therefore, $\{k \in \mathbb{N} : \gamma(x_k, \xi) \geq D(\beta_k)\} \subseteq \mathbb{N} - M \in I$. Consequently $OI - \lim x_n = \xi$.

The following example ensures that for an ideal I a sequence $\{x_n\}$ in L, $OI - \lim x_n$ and $OI^* - \lim x_n$ may not be equal.

Example 3.16. Let $N_p = \{p, p^2, p^3, ...\}$, where $p \in P$, the set of all prime numbers and $N_1 = \mathbb{N} - \cup_{p\in P} N_p$ Then $\mathbb{N} = \cup_{j=1, j\in P} N_j$ where each N_j is infinite and $N_i \cap N_j = \phi$ for $i \neq j$. Consider $I = \{\ A \subset \mathbb{N} : A \textit{ intersects only a finite number of } N_j\text{'s}\ \}$.
Let L has an accumulation point ξ in L. Then there exists a sequence $\{x_n\}$ in L so that $\lim_{n\to\infty} x_n = \xi$.
Using Lemma 3.7 we can choose a sequence $\{\alpha_n\}$ in L with $\alpha_n \downarrow \theta$ so that $\gamma(x_n, \xi) < D(\alpha_n)$ for all $n \in \mathbb{N}$.
Define a sequence $\{y_n\}$ in L with $y_n = x_j$ if $n \in N_j$, where j is either 1 or a prime number. We assert that $OI - \lim y_n = \xi$. If not, then for each $\beta = \{\beta_n\}$ in L with $\beta_n \downarrow \theta$, $A(\beta) = \{n \in \mathbb{N} : \gamma(y_n, \xi) \geq D(\beta_n)\} \notin I$. i.e. $A(\beta)$ intersects infinite number of N_j's. Then there exists a subsequence $\{p_n\}$ of prime numbers such that $\gamma(y_n, \xi) \geq D(\beta_n)$ when $n \in N_1 \cup N_{p_1} \cup N_{p_2} \cup N_{p_3} \cup ....$
Since N_{p_1} is infinite, there exists a natural number $n = q_1 \in N_{p_1}$ such that $\gamma(x_{p_1}, \xi) \geq D(\beta_{q_1})$.
Further N_{p_2} is infinite, so there exists a natural number $n = q_2 \in N_{p_2}$ with $q_2 > q_1$ such that $\gamma(x_{p_2}, \xi) \geq D(\beta_{q_2})$.
Continuing this process we can construct a subsequence $\{q_n\}$ of natural numbers such that $\gamma(x_{p_n}, \xi) \geq D(\beta_{q_n})$ for all $n \in \mathbb{N}$ but in particular if $\beta_{q_n} = \alpha_{p_n}$, this contradicts the choice of $\{\alpha_n\}$.

Now if possible let $OI^* - \lim y_n = \xi$. Then there exists a set $M = \{m_1 < m_2 < m_3 < ...\} \in F(I)$ such that $\lim_{k\to\infty} x_{m_k} = \xi$ with respect to the metric γ. Using Lemma 3.7 we can have a sequence $\{\beta_n\}$ in L with $\beta_n \downarrow \theta$ such that

$$\gamma(x_{m_k}, \xi) < D(\beta_{m_k}) \text{ for all } k \in \mathbb{N} \qquad .....(1)$$

Let $H = \mathbb{N} - M$, then $H \in I$ and there exists prime numbers p_1, p_2, ..., p_r such that $H \subseteq N_1 \cup N_{p_1} \cup N_{p_2} \cup ...N_{p_r}$. Thus, $N_{r+1} \subseteq \mathbb{N} - H = M$ and $n \in N_{r+1}$ implies that $y_n = x_{r+1}$ for infinitely many n. i.e. for infinitely many m_k, $\gamma(y_{m_k}, \xi) = \gamma(x_{r+1}, \xi) > 0$ which contradicts the relation (1) since $\beta_n \downarrow \theta$ and D is monotone increasing. Hence $OI^* - \lim y_n \neq \xi$.

Theorem 3.17 Let I be an admissible ideal of subsets of $\mathbb{N}$ and I has the AP-property. Then for a sequence $\{x_n\}$ in (L, γ), $OI - \lim x_n = \xi$ if and only if $OI^* - \lim x_n = \xi$, $\xi \in L$.

Proof: Since $OI - \lim x_n = \xi$, then we can choose a sequence $\{y_n\}$ of distinct elements in L with $y_n \downarrow \theta$ such that $\{n \in \mathbb{N} : \gamma(x_n, \xi) \geq D(y_n)\} \in I$.
Consider $A_1 = \{n \in \mathbb{N} : \gamma(x_n, \xi) \geq D(y_1)\}$ and
$A_n = \{k \in \mathbb{N} : D(y_n) \leq \gamma(x_k, \xi) < D(y_{n-1})\}$, for $n \geq 2$.
Clearly A_i's are pairwise disjoint. By AP-property there exists a sequence of subsets $\{B_n\}$ such that $A_i \Delta B_i$ is finite for all $i \in \mathbb{N}$ and $B = \cup_{i=1}^{\infty} B_i \in I$
Let $M = \mathbb{N} - B = \{m_1, m_2, m_3, ...\}$.
For $\varepsilon > 0$ we choose the smallest positive integer $k \in \mathbb{N}$ such that $D(y_{k+1}) < \varepsilon$. Then $\{n \in \mathbb{N} : \gamma(x_n, \xi) \geq \varepsilon\} \subseteq \cup_{i=1}^{k+1} A_i$.
$A_i \Delta B_i$, $i = 1, 2, 3, ..., k+1$ are all finite sets and so there is some $m \in \mathbb{N}$ such that $\cup_{i=1}^{k+1} B_i \cap \{n \in \mathbb{N} : n > m\} = \cup_{i=1}^{k+1} A_i \cap \{n \in \mathbb{N} : n > m\}$.
If $n > m$ and $n \notin B$ then $n \notin \cup_{i=1}^{k+1} B_i$ and this implies that $n \notin \cup_{i=1}^{k+1} A_i$. Then $\gamma(x_n, \xi) < D(y_{k+1}) < \varepsilon$.
Therefore, $\gamma(x_n, \xi) < \varepsilon$ for $n > m$ and $n \in M$ and hence $\lim_{n\to\infty} x_{m_n} = \xi$ and consequently $OI^* - \lim x_n = \xi$.
The converse follows from Theorem 3.15.

4 Conclusion

In this paper, two new concepts, namely the concepts of OI-convergence and OI^*-convergence in a linearly ordered additive system have been introduced and investigated. In this investigation we have also shown by an example that OI-convergence and OI^*-convergence need not be equivalent. Further we have introduced the idea of OI-bounded sequences and investigated some basic properties. The present paper also contains a generalization of the results of the papers [3] and [9]. In this perspective we think that these results could provide a more general frame work for the investigation on convergence of sequences with respect to order.

Acknowledgement

The authors are grateful to the reviewers for careful reading and making some useful corrections which improved the presentation of the paper.

References

[1] C. D. Aliprantis, Kim C. Border, *Infinite Dimensional Analysis*, Springer Berlin Heidelberg, (Third Edition), New York, (2006).

[2] G. Birkhoff, *Lattice Theory*, Amer. Math. Soc. Colloq. Pubi. vol. XXV (First Edition), New York,(1940).
[3] D. K. Ganguly, Bablu Biswas, *Order statistical convergence in a metric space*, Investigations in Mathematical Sciences, 4(2), 2014, 11-23.
[4] P. Das, S. Dutta and S. K. Pal, *On I and I* Equal Convergence and an Egoroff-type theorem*, MATEMATIKI VESNIK, **66**, 2 (2014), 165177.
[5] H. Fast, *Sur la convergence statistique*, Colloq. Math., **2** (1959), 241–244.
[6] J. A. Fridy, *On statistical convergence*, Analysis, **5**(no.4)(1985), 301–313.
[7] O. Frink, *Topology in lattices*, Trans. Amer. Math. Soc., **51**(1942), 569–582.
[8] P. Kostyrko, M. Macaj, T. Salat, M. Sleziak, *I-convergence and External I-limit Points*, Math. Slovaca, **55**(2005), 443-464.
[9] P. Kostyrko, T. Salat, W. Wilczynski, *I-convergence*, Real Anal. Exchange,**26(2)**,(2000), 669-688.
[10] H. I. Miller, *A measure theoritical subsequence characterization of statistical convergence*, Trans. Amer. Math. Soc.,**374**(no.5)(1995), 1811–1819.
[11] I. Niven, H. S. Zuckerman, *An Introduction to the Theory of Numbers*, John Wiley & Sons, New York, USA, 4-th edition, Chapter-11, 1980.
[12] B. C. Rennie, *Lattices* , Proc. Lond. Math. Soc. Series **52**(no.2)(1951), 386–400.
[13] T. Salat, B. C. Tripathy, M.Ziman, *On I-convergence field*, Ital. J. Pure Appl. Math., 17((2005), 45-54.
[14] I. J. Schoenberg, *The integrablity of certain functions and related summability methods*, Amer. Math. Monthly, **66(no.** 5) (1959), 361–375.
[15] H. Steinhaus, *Sur la convergence ordinaire et la convergence asymptotique*, Colloq. Math., 2(1951), 73–74
[16] B. K. Tripathy and B. C. Tripathy, *On I-convergent double sequence.*, Soochow J. of Math., **31**(no.4)(2005), 549-560.
[17] B. C. Tripathy, B. Hazarika, *On I-monotonic and I-convergent sequences.*, Kyungpook Math. J., **51**(2) (2011), 233-239.
[18] B. C. Tripathy, B. Hazarika, *I-convergent sequence spaces defined by Orlicz function*, Acta Math. Appl. Sin., **27**(1) (2011), 149-154.
[19] B. C. Tripathy, M. Sen, S. Nath, *I-convergence in probabilistic n-normed space*, Soft Comput., **16** (2012), 1021-1027, DOI 10.1007/s00500-011-0799-8.
[20] B. C. Tripathy, A. J. Dutta, *On I-acceleration convergence of sequences of fuzzy real numbers*, Math. Modell. Anal., **17**(4) (2012), 549-557.
[21] L. R. Wilcox and M. F. Smiley, *Metric Lattices*, Ann. of Math., **40**(no.2)(April 1939),309–327.

Bablu Biswas is an Assistant Professor in Mathematics and a research scholar. He published 6 research papers in reputed national and international journals. He also delivered lectures in some national and international seminars. His research directions are in the area of descriptive set theory, sequence spaces, series and measure theory. He has completed his M.Sc. degree in Mathematics from the Department of Pure Mathematics, University of Calcutta and is a life member of Calcutta Mathematical Society, Kolkata.

D. K. Ganguly is a retired Professor of the Department of Pure Mathematics, University of Calcutta, India. He received Ph. D degree in Pure Mathematics from University of Calcutta. He pursued Post-Doctoral Research work in the University of California, Santabarbara, USA. He has undergraduate and postgraduate teaching experiences more than 44 years. His research interests are in the area of Real Analysis, Descriptive Set Theory, Measure Theory, Fixed Point Theory, Function Spaces, Multi-valued Functions and Selection Theory and also Integration Theory. He has published 80 research papers in reputed International and National Journals. He is referee and member of the Board of Editors of some International and National journals. He is at present a Vice President of Calcutta Mathematical Society which is one of the oldest Societies in the world. He is a member of some learned Societies. He visited various universities and institutions in abroad, namely, USA, England, Ireland, Canada, Switzerland, Slovakia, Hong Kong, Thailand, Indonesia on academic invitation. He has produced so far 10 Ph. D students. He is member of several academic committees in some universities in India.

15/5/23